The Royal Aeronautical Society

(FOUNDED 1866)

with which is incorporated The Institution of Aeronautical Engineers

No. 242. FEBRUARY, 1931. Vol. XXXV.

PROCEEDINGS

SECOND MEETING, FIRST HALF, 66TH SESSION

The second meeting of the 1930-31 Session of the Royal Aeronautical Society was held on Thursday, October 23rd, 1930, at 6.30 p.m., in the Lecture Hall of the Royal Society of Arts, 18, John Street, Adelphi, W.C.2, at which Mr. F. W. Meredith, B.A., read a Paper on "Air Transport in Fog." The President, Mr. C. R. Fairey, was in the chair.

The PRESIDENT: Mr. Meredith has been engaged in experiments in fog flying. Fog was the great difficulty encountered by all forms of transport, including that by boat, train, or car. Those vehicles, however, could stop when they wished, but an aeroplane could neither see nor stop, so it appeared to be a hundredfold more difficult problem to solve. On the other hand, it was true that it was possible for it to move in the air with correspondingly less risk of collision and it had one sole advantage—it could rise clear of the fog and is usually only affected by it in the take-off or landing, but in every other respect the aeroplane was very considerably handicapped in fog compared with any other vehicle. Mr. Meredith had attacked the problem in a way which would appeal to the aircraft designer; he did not complain of the aeroplane at all; he took it as it was and merely sought to endow it with sense of hearing, touch, equilibrium and direction not possessed by any other vehicle. He had been engaged on fine work. He did not pretend to have found a complete solution of the problem, but he had provided for them a record of some most impressive research and experiment and the development of systems and apparatus which had culminated in the fact that an aeroplane had been successfully managed in a fog, flying for a period in the upper air and landing in a fog 90 feet thick.

AIR TRANSPORT IN FOG

BY

F. W. MEREDITH, B.A.

It is not possible to go fully into all the aspects of this problem in the time at my disposal. I am accordingly confining myself to a discussion of the feasibility of operating heavier-than-air craft in fog with reference to the instrumental equipment required. The meteorological aspects were dealt with in a paper by Captain Entwistle two years ago.

In order to obtain a correct perspective on the problems of flying in fog it is necessary to realise that many cases of impaired visibility, which would not normally be regarded as fogs, may seriously increase the risks of flying. This reduction of visibility may be caused by fog proper or by haze, drizzle, rain, snow or dust. The problem I wish to consider then is that of flying, navigating and landing when the visibility is such, that flight would be impossible or unduly dangerous without special equipment.

Until recently it was generally regarded as impracticable to continue flying in a heavier-than-air craft when the pilot was unable to see a sufficient distance to identify his position and to avoid such obstructions as buildings, trees or hills. Even if it is known that the destination is clear of fog, pilots have a reasonable dislike of losing sight of the ground and prefer to fly low until the visibility becomes so bad that they are forced to land. This dislike is reasonable, because to fly long distances out of sight of the ground entails a serious risk in the event of the failure of engines, instruments or wireless, as the pilot may be unable to regain a view of the ground in time to choose a position for a forced landing; even if the engine power is still available, he may not be able to avoid obstacles while looking for the ground.

Recently much attention has been devoted, in this country and abroad, to the problem of landing in fog, and many people have questioned the necessity for this policy of " safety first." It has been suggested that, if certain technical difficulties could be surmounted, aircraft would at once defeat the greatest enemy of all transport. Others, feeling perhaps that such an opinion displays by its arrogance the youthfulness of flying, have reminded the optimists that even the birds of the air prefer the ground in fog.

Now there is a vast difference between the ability to fly in fog and the advisability of doing so. This difference is apt to be overlooked when one is concentrating on the technical difficulties of the problem. Let us then take as our criterion, that flying in fog is only justifiable if it does not materially increase the risk of accident.

In certain circumstances, for example, in crossing the Atlantic, one may be compelled to fly in fog as the lesser of two evils. The ability to fly and land in fog then becomes the only means of avoiding a disaster. In these circumstances, the reliability of the equipment need not approach perfection. It is not suggested that it need not be the highest attainable, but, on the principle of " any port in a storm " provision should be made for fog, even if the best that can be done is but to reduce the risk of an accident. For the same reason, economic considerations of the loss of paying load must be secondary.

When, however, one considers the policy of flying deliberately in fog, the question of the risk assumes a vastly different complexion. It then becomes essential that the methods employed should be infallible and that the equipment, upon which one depends, should be comparable in reliability with a railway signalling system. Now this standard of reliability is very much higher than that normally required of aircraft equipment and is, moreover, not likely to be attained for some time. Recently two very able and gallant men set out from this country to fly non-stop to the Cape and lost their lives in a crash near Tunis. The official report on the accident suggests that it was probably caused by a failure of the altimeter which misled the pilot as to the height of the aeroplane while they were approaching the hills in bad visibility. Yet if one were asked to name the most reliable instrument used on an aeroplane, one would probably choose the altimeter. Without wishing to detract from the valuable work which has been done in adapting wireless for aircraft use, I may perhaps be permitted to say that wireless is still a tricky ally, even for ground use. Engine reliability has improved out of recognition since the days when one expected a forced landing whenever one flew, but I doubt whether anyone would even now be prepared to fly in fog if an engine failure involved a descent by parachute. It is true that this risk is materially reduced if the aeroplane is capable of flying without overtaxing the remaining engines when one has failed. Instruments, as well as engines, may be partially insured against failure by redundancy, but there must be a limit to the special load which an aeroplane may carry to provide against an occasional risk if that risk be avoidable.

It is clear then, that one must decide whether it is practicable to avoid flying in fog before deciding to provide for such a contingency. I suggest that the

answer to this question will depend upon the duration of the flights, the climatic conditions and above all on the adequacy of the system of fog warning in operation. Fog, as far as I can ascertain, does not form rapidly enough to catch one unawares, if its beginning or probability is instantly reported over a wide area, unless one has to cover large distances between landing grounds. This is, I think, true even with the wider significance which I am attaching to the term fog. If this view be accepted, it follows that, on such a route as Croydon-Paris, fog flying is avoidable and the risk, the extra cost of carrying the necessary equipment, and the costs of the ground organisation, must be offset against the advantages of maintaining a regular service. As a compromise it might be desirable to maintain a restricted service in fog with specially equipped aeroplanes. On a long distance route, however, if there are long stretches devoid of emergency landing grounds, fog flying may be an unavoidable contingency and safety may require that provision should be made for this emergency, regardless of the cost, if the service is to be maintained.

With regard to the special equipment required, the problem naturally divides itself into two parts, *i.e.*, flying " blind " to one's destination, and landing. I will deal first with the relatively simple problem of " blind " flying.

Any pilot with a little practice can fly an aeroplane in fog by the aid of the airspeed indicator, the compass and a turn indicator. It requires close attention and is fatiguing even in calm air, but in bumpy air it proves very exhausting. For many years automatic control of the rudder by a servo motor, itself controlled by a gyroscope, has been available and this is capable of flying a normal aeroplane with only occasional direction by the pilot, who is thus freed from the strain and allowed to devote his attention to the navigation and other matters. Experience has shown that, provided the aeroplane is stable longitudinally and has a normal amount of dihedral, the other controls may be ignored even in rough weather. The aeroplane may wallow, but its stability is assured. Gyroscopic apparatus capable of controlling both the rudder and elevators has now been developed and tested fairly extensively in the Service. I must ask to be excused giving any details of this equipment or entering into discussion on the subject of automatic controls as foreign patents are pending which might be invalidated by premature publication. The equipment has been released for civilian use, however, and Messrs. Imperial Airways are contemplating testing it on one of their longer routes. The apparatus weighs, in its present form, about 80lb. as installed in a large aeroplane, but I anticipate that this figure could be substantially reduced with further development.

Now the important lesson to be learnt from the experimence gained with automatic controls, is that, for maintaining straight flight, it is unnecessary to observe either the horizontal attitude or the lateral bubble, and it is unnecessary to move the ailerons. Many pilots have already acquired this knowledge and developed a simple method of flying by turn indicator which considerably reduces the strain. They hold the controls in such a manner as to avoid the application of any load on the ailerons and plant their heels firmly on the floor to avoid over-correction on the rudder.

It is a matter for regret that many pilots demand, and most instrument designers provide, a lateral bubble as part of the blind flying equipment (attached to the turn indicator or any alternative instrument) although no one can say precisely what the pilot is expected to do with the information so provided. To make matters worse they frequently call the lateral bubble or pendulum a " bank indicator." Now a pilot, flying normally, moves the ailerons to correct bank, the bank being observed by reference to the horizon. To apply aileron to correct the lateral bubble, is, however, merely equivalent to having more dihedral unless the pilot is possessed of superhuman powers of analysing the motion and anticipating; more often he lags behind the instrument and only makes matters worse.

Moreover, the unwanted yawing moment of the ailerons creates further disturbances which must be corrected by the rudder.

Lieutenant Doolittle, who was the pilot of the Guggenheim Fund for the promotion of Aeronautics, suggested, and had provided by the Sperry Company, a gyroscopic pitch and bank indicator which not only allows of the proper use of ailerons and elevators, but renders a turn indicator redundant. He argued with justification that a pilot, deprived of the horizon, would be most assisted by the provision of an artificial horizon. Fig. 1, taken from a pamphlet of the Guggenheim Fund entitled " Equipment used in Experiments to solve the Problem of Fog Flying," shows five views of the dial of this instrument. It will be seen that a picture of the aeroplane, flying away from the observer, is affixed to the cover glass and banks with the aeroplane while the white line representing the horizon remains horizontal, being attached to the gimbal system of the gyroscope. In addition the longitudinal attitude is shown by the rise or fall of the horizon line in sympathy with the true horizon.

For the reasons which I have given, I prefer an instrument which provides the pilot with precisely the same information as the two axes automatic pilot uses, namely, deviation in pitch and yaw. Flying by such an instrument is equivalent to flying on a point, such as the moon or a star when near the horizon. Such an instrument has been designed at Farnborough and is shown in Fig. 2. When the attitude of the aeroplane is correct, a white circular disc indicating the position of the gyroscope rotor is central behind a circular black mark on the cover glass. Any deviation is indicated by the emergence of a crescent of a white disc, and the pilot must deflect the nose of the aeroplane, as represented by the white disc, towards the datum, as represented by the black mark. Very little practice is needed to acquire the art of maintaining straight flight by the use of this instrument. A person who had no experience of piloting was able, after about five minutes practice, to fly straight through a bank of clouds extending for about five miles. This instrument has a safety device which prevents excessive displacement of the gyroscope should the pilot intentionally or accidentally turn beyond the limit of 12° provided for on the face of the instrument. An instrument, functionally similar but electrically driven, has just been produced by Messrs. S. G. Brown and is illustrated in Fig. 3. Both these instruments possess the useful property that the gyroscope will precess slowly towards the central position if the pilot flies with it displaced. Thus delicate corrections may be applied to either the pitch or yaw datum.

There are many other types of instrument which make it possible to maintain straight flight when deprived of the natural horizon, but it is perhaps clear that this part of the problem is already solved and that reliability must be the determining factor in selecting the equipment for the purpose. Redundancy of such equipment, such as the addition of a turn indicator whether the primary instrument be an automatic pilot, an artificial horizon, or a pitch and yaw indicator, is, however, essential.

To be able to fly straight is, however, of little advantage in fog unless one reaches one's destination. Dead reckoning is clearly of no use unless one is sure of emerging into clear air before reaching one's destination. Wireless in some form is therefore absolutely essential. To distinguish this part of the problem from the landing, I will assume that one requires to arrive within a mile or so of the landing ground. The radio methods known for this purpose come under four headings:—

(*a*) Direction finding on the ground.
(*b*) Direction finding in the aeroplane.
(*c*) Track indicating systems.
(*d*) Rotating beacons.

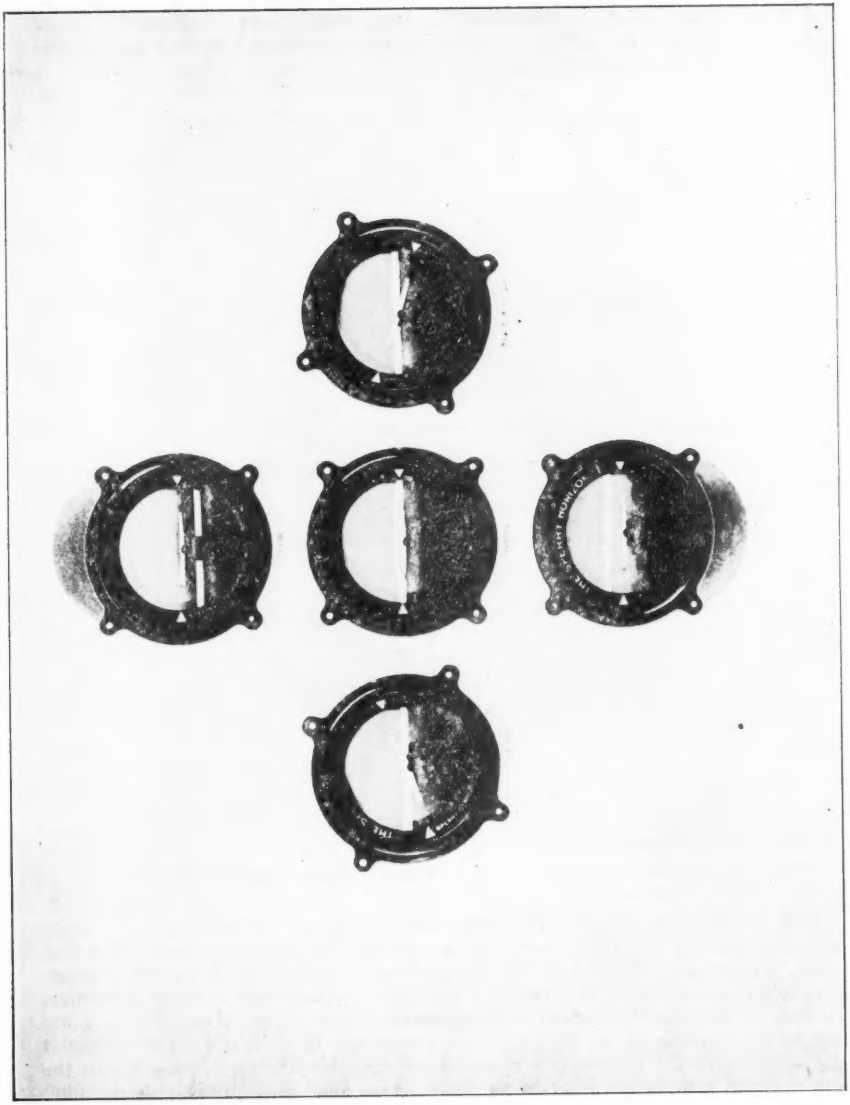

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FIG. 1.

The first of these has been in use in this country for some years and is well known. The position has to be determined on the ground and transmitted to the pilot although the " homing " bearing may be given quickly from the station at the destination. The system is adequate, if cumbrous, and is generally sufficient if the visibility at the aerodrome is such that a very accurate fix is not required for the actual landing. It is in any case sufficient for the part of the problem we are now considering.

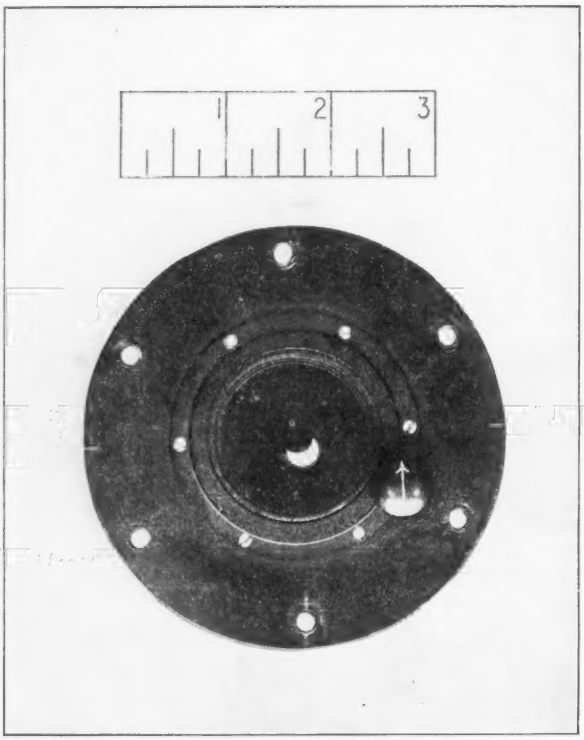

FIG. 2. *Crown Copyright reserved.*

For direction finding in the aeroplane, the signal strength on two aerial systems possessing different directional properties is compared and the pilot flies to keep the aeroplane headed for the transmitter. Usually the indication is aural, but it is quite practicable to produce a continuous visual indication by a pointer. One form of such an instrument is being developed at Farnborough, but it is too early to give details of its design. The advantage of such a system is that it alone supplies all the information required for directional control, whereas all the other systems require the pilot to fly by compass and turn indicator or the like, as well as checking his position by the radio device.

Of track indicating systems there are several, but it is sufficient to mention the equi-signal beacon which has been extensively used in the U.S.A. This device was the central feature of the fog flying experiments of the Guggenheim Fund. In this system two signals are transmitted in such a manner that each is at a

maximum strength on a particular bearing from the transmitter, whereas the two are of equal strength on the bisecting bearing. As the polar diagram of signal strength for each signal is a figure of 8, it is apparent that the signals will be equal on the four cardinal points of the system. The indication may be aural or visual; in the former case the signals key together to form either a continuous note or a series of continuous dashes when the receiver is on one of the four equi-signal " beams," whereas characteristic morse letters predominate when one or other signal is stronger; in the latter case two tuned reeds are energised by the note modulation of the two signals and the pilot must set his course to maintain equal amplitudes on the two reeds. These systems possess the outstanding advantage that the normal radio receiver is used without special equipment except for the box of reeds which is simple and weighs but a few

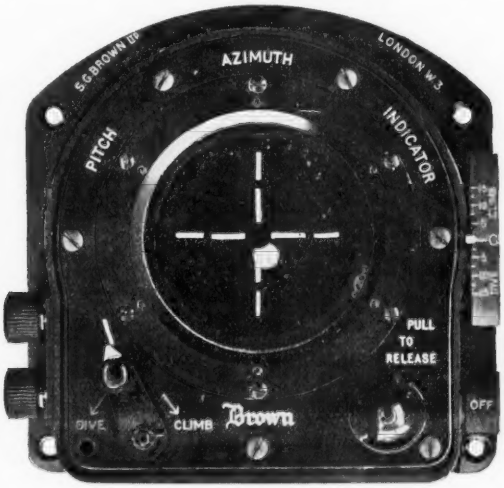

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FIG. 3.

ounces. One must not assume, however, that steering an aeroplane to follow the track of an equi-signal beam is a simple and straightforward matter. The pilot must be sure which of the four beams he is in, estimate his drift and fly on such a compass course, by the aid of a turn indicator, as to follow the beam. In the work of the Guggenheim Fund, Lieutenant Doolittle took off, flew a total distance of about 20 miles and landed again with a hood over his cockpit, using such a beam. It is stated, however, that he was provided, in addition, with a conventional compass, an earth inductor compass, a free gyroscope as an azimuth indicator free from the periodic and tilting errors of compasses, a Sperry artificial horizon, and a turn indicator.

Rotating beacons are capable of giving the pilot his bearing from the beacon on which his receiver is tuned. In its simplest form the polar diagram of the field contains a sharply defined minimum, *i.e.*, the diagram is a " cardiac." This field is rotated at a regular speed and a characteristic signal marks the time when the silence bearing is north. By taking the time between the north signal and the moment when the signal is a minimum, the pilot may deduce his bearing. A stop-watch graduated in degrees may be used for this purpose.

This system also requires no special equipment in the aeroplane and in addition gives the correct bearing without ambiguity, wherever the aeroplane may be, as long as it is in range of the transmitter. If two such beacons can be received, the position may be accurately determined.

I do not wish to say which of these radio systems is to be preferred; there is something to be said for each and for more complex systems which have been proposed, but there can be no doubt that this part of the fog flying problem does not present serious difficulty in the present state of knowledge.

I come now to the most difficult part of the problem—landing. This demands a very high accuracy in fixing one's position in order to land within the limited space provided in an ordinary aerodrome. One solution has already been provided by the Guggenheim Fund. By the use of the equipment I have described,

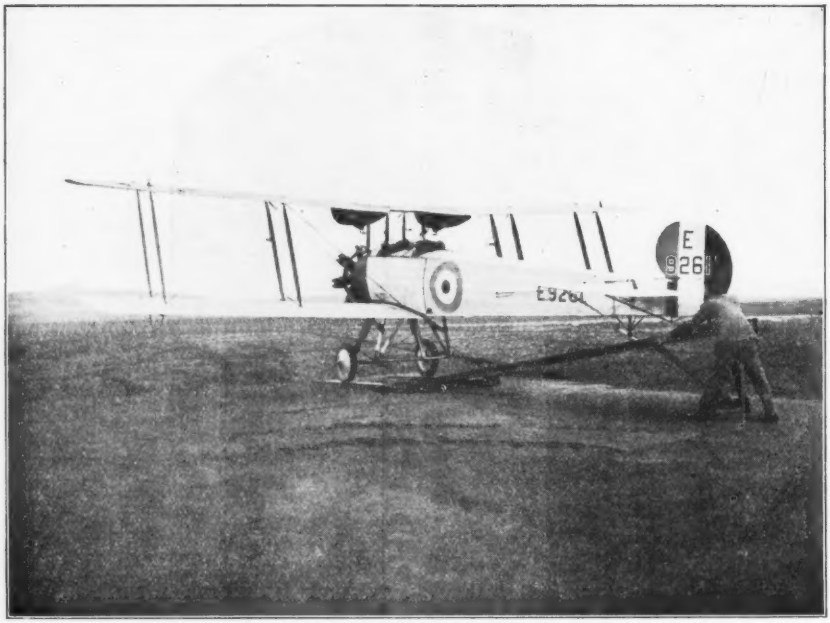


FIG. 4. *Crown Copyright reserved.*

Lieutenant Doolittle was able to land " blind " at Mitchel Field. A specially accurate barometric altimeter enabled him so to fix his altitude that he could begin his descent when he was vertically over the transmitter. A special " marker " beacon had been designed to give the signal for the descent, but it was not satisfactory and it was found to be practicable to detect when the aeroplane was vertically over the main beacon by the sudden drop in signal strength. No attempt was made at flattening out, reliance being placed on a special undercarriage and regulation of the rate of descent.

The " leader cable " provides another possible solution of the landing problem, but in practice it is difficult to follow, more particularly when it is curved and when the air is " bumpy." Moreover, this solution requires, at present, what may be regarded as a prohibitive amount of special equipment both in the aeroplane and on the ground. It cannot be too strongly emphasised that the aeroplane

equipment for fog flying should consist in the main of instruments normally carried and in daily use.

It is probable that the complete solution of the fog landing problem will depend upon the ordinary wireless receiver for leading the aeroplane over the landing ground and for fixing the moment when the engine should be throttled back, the height being fixed by a barometric altimeter. Much has been said about other methods of measuring height such as timing echoes, acoustic or radio, or measuring variation in the electrical capacity. It is safe to say that the barometric method is capable of giving a much higher accuracy than is required for fog landing, provided that the ground barometric reading be communicated to the pilot. I am not, of course, contemplating a forced landing

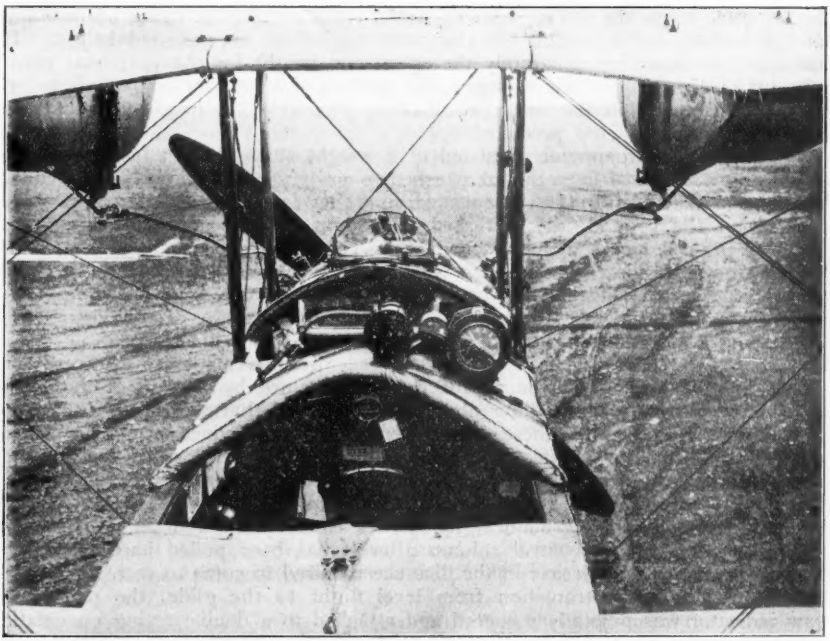


FIG. 5. *Crown Copyright reserved.*

without the co-operation of ground personnel. That the flattening out manœuvre presents no difficulty, will appear from the following account of some recent experiments which have been conducted at Farnborough.

These experiments were intended to investigate the practicability of landing through a fog of limited height, guided by a balloon moored above the fog. The work included a determination of the magnitude of the errors of the positioning of the landing and a demonstration with a standard Lynx Avro, by Flight Lieutenant Oddie and Mr. Sudworth, in an actual fog. It was necessary to develop safe methods of taking off and of flattening out which could be used even if the ground were invisible.

Taking off presented no difficulty whatever with the assistance of the pitch and yaw indicator. It and a simple form of ground proximeter, to be described presently, were the only special instruments carried on the aeroplane. For

taking off the tail skid was placed on an inclined ramp and secured by a wire to a bolt which could be withdrawn by a person on the ground after a signal from the pilot (Fig. 4). The attitude of the aeroplane on the ramp was chosen to correspond with the attitude on a slight climb. By releasing the gyroscope in this position, the pilot was provided with a datum in lieu of the horizon, as well as the means of keeping dead straight.

The flattening out manœuvre might have been rendered unnecessary had a special undercarriage been designed to absorb the downward momentum of the glide. Such a solution is, however, not likely to recommend itself to nervous passengers and it was known from previous work that a semi-automatic landing could be achieved by the aid of a ground proximeter. It had been shown that an aeroplane approaching the ground at an assigned speed (estimated at about 20 per cent. above the stalling speed), would reach the ground flying horizontally at the stalling speed, if full elevator were applied at an assigned height. It remained to determine accurately the speed and height for the particular aeroplane we were using. This was done by analysis of accelerometer records taken in the aeroplane while the manœuvre was repeated at a safe height from various gliding speeds near the predicted speed.

The ground proximeter consisted of a weight suspended 13 feet below the wheels by a length of linen thread attached to a trigger controlling lamp circuits. As long as the weight was suspended, a green lamp showed and when the weight hit the ground, the increased tension in the thread switched on two independent red lamps with independent dry batteries and circuits. The indicating lamps are shown in Fig. 5, a view of the two special instruments grouped with the airspeed indicator in a convenient position for the pilot. Two feet on the length of the string was allowed for the pilot's lag in responding to the signal.

About 100 landings have now been made by this method, the necessary precision of flying during the approach being attained by the aid of the pitch and yaw indicator. Three different pilots have found that there is no difficulty in the manœuvre unless there is sufficient wind to produce a marked wind gradient. To meet this difficulty, which always produces a loss of flying speed as the aeroplane approaches the ground (provided, of course, it is landing up wind), it is desirable to add a couple of miles an hour to the speed. Slight excess speed when the ground signal occurs is easily allowed for by a slight easing forward of the control column after it has been pulled hard back and the only result is an increase in the distance required to come to rest.

To facilitate the transition from level flight to the glide, the pitch and yaw indicator was pivotally mounted and attached to a double acting pneumatic ram which was controlled by a tap placed beside the throttle lever. Thus, once the stop on the ram had been correctly adjusted, the pilot was able to rotate the instrument in pitch through the gliding angle as he closed the throttle and by flying to keep the indicator central, he placed the aeroplane at the correct attitude for the glide. Any error in the datum of the indicator was apparent on the airspeed indicator and the pilot was able to correct it by flying with the pitch indicator slightly displaced.

To start the glide from the balloon, the pilot proceeded as follows. The assigned course was set on the compass before leaving the ground. The pilot first ascertained the engine speed required to fly level at the assigned speed, and, having noted his height when alongside the balloon, he arranged to fly away from the aerodrome on a course parallel to the assigned course and sufficiently far away to allow for the turn of 180°. After flying level on this course for about a mile and a half, he turned the aeroplane and flew straight towards the balloon until the compass had settled after the turn. He then corrected his course by the compass when close to the balloon and released the gyroscope of the pitch and yaw indicator. He then manœuvred until he was

alongside the balloon (moving the throttle if necessary to adjust his height) and simultaneously closed the throttle and moved the tap to alter the pitch datum. After that, he merely had to keep the pitch and yaw indicator central until he received the signal to pull the control column hard back.

The gliding angle was calculated from the measured rate of descent. After allowing for the wind, the balloon height and compass course were adjusted so that the weight should strike the ground at an assigned position in the centre of the aerodrome. The wind was measured in some cases by the standard meteorological method, and in other cases it was estimated from the angle of the balloon cable. A table of balloon heights and compass courses was prepared, allowing also for the horizontal displacement of the balloon from the winch. Thus, once the wind had been estimated, little time was required for the final adjustments.

After fifteen practice landings had been made with the balloon at heights varying from 370 to 420 feet according to the wind, it was considered safe to take off and land in a low fog, as long as an observer on the top of the balloon shed was clear of the fog; this precaution was necessary to ensure that the balloon, which was too small to lift a man, was above the fog. On the morning of 18th June, 1930, the weather conditions were suitable at sunrise and five landings were made through a fog estimated as 90 feet in depth. The visibility in the fog was variable, but the pilot was not able to observe the ground after entering the fog.

The maximum errors of all the landings made from this height were 95 yards in range and 50 yards in line. A few tests were made with the balloon at a height of 750 feet. These indicated that the errors were about double those with the balloon at 400 feet. It would appear, therefore, that on any reasonably large aerodrome, landings could be made by this method through a fog extending up to about 700 feet, provided that the ground personnel could ascertain that the balloon was in clear air. This could be ensured by using a man-carrying balloon, or by providing instruments capable of giving the necessary information. May I suggest that the design of such instruments would be an interesting and fruitful occupation for persons of an inventive turn of mind who are interested in the problem of fog landing?

I have described this recent work at some length, because it would appear that this solution would deal with a large proportion of the fogs proper which impede flying in this country, and also because the method can be applied with little modification with any radio device for fixing the start of the glide instead of the balloon.

As a safeguard it might be desirable to observe the position of the aeroplane from the ground and communicate a warning to the pilot if he were obviously landing in the wrong place. Such a system might give added confidence to the pilot by allowing him to share the responsibility with ground personnel who would not be subjected to so severe a strain. It is hardly likely that radio could be used for this purpose, since it would necessitate simultaneous transmission and reception in the aeroplane. Acoustic means might be used; possibly the necessary accuracy of location could be achieved by the unaided human ear if the observers were suitably placed. It would, however, be exceedingly difficult to check the altitude at any time and it is possible that reliance would have to be placed on duplication in the aeroplane of the means for measuring this quantity. This might be regarded as a justification for developing an alternative form of altimeter. The balloon method is of course free from this complication.

It has been suggested that fog flying will never be practicable on account of the risk of collision. Although there is little prospect of the development of equipment capable of giving adequate warning of the approach of another aeroplane in a thick fog, any development which enables aeroplanes to go up in bad visibility, instead of operating at a common height (*i.e.*, the maximum

height from which their pilots can view the ground), would materially reduce the risk of collision. It would be advisable to reserve specific heights for each line of traffic and thus reduce the risk to that of one aeroplane overtaking another, when the relative speed would be so low as to give the overtaking pilot a reasonable chance of seeing the other aeroplane in time. The risk could be still further reduced, if the ground personnel controlled by radio the height of each aeroplane. This might lead to great complication if the traffic coming into an airport were really congested, but it would clearly be impracticable to maintain such a service in fog, since time would have to be allowed to clear each aeroplane off the landing ground, before permitting the next aeroplane to land.

It will be noticed that the bulk of the special equipment required on the aeroplane is such that it might reasonably be carried for normal flying. The automatic pilot will probably be used for all long range flights in the near future, not only because it relieves the pilot of the strain of flying, but also because it should permit of a reduction of one in the crew required. I do not wish to specify the allocation of duties on an automatically controlled aeroplane, but clearly the pilot is free to attend personally to the navigation and any member of the crew can stand by to take charge in the event of a failure of the automatic controls while the pilot rests. In any case it is now generally recognised that efficient blind flying instruments are an essential part of any aeroplane's equipment. Wireless is no less essential. Thus the special equipment required for the densest fog should not amount to more than the sensitive aneroid, the reed indicator (if, for example, the equi-signal beacon system be used) and the ground proximeter.

In conclusion, I would say that air transport in fog is immediately practicable, but that it involves an element of risk which will continue until all the equipment required has been proved by many years of actual use. In the meantime the policy should be to press on with the development of the necessary instruments, to equip aeroplanes for landing in fog when they are travelling such distances that they may be enveloped before reaching a possible landing ground, but to avoid flying in fog whenever possible.

My neglect of the subjects of fog dispersion and of infra red rays as a means of seeing through fog, is an indication of my view of the utility of such suggestions in the present state of physical knowledge.

Discussion

The President: He would like to be enlightened upon one or two points in the paper. He was disappointed to learn after many years that the bubble was of no use. That only showed on what false hopes many people had pinned their faith. In pre-war days, when learning to fly, they were taught to follow the bubble with the top of the stick. Apparently they were doing wrong the whole time. He would like to know how much faith Mr. Meredith placed in directional wireless. He had a vivid recollection of the late Commander Rogers, who was marooned for eight or nine days in the Pacific with a big flying boat, telling him of his experience of flying by radio bearings. On the second day of the flight the Commander was lost and he spent much time flying up and down the radio bearings without finding a trace of any cruiser. Finally, after he had run out of petrol he put on his receivers and heard one of the cruisers inquiring of the other, " Why does not that darned fool use his radio?"

Mr. Meredith's paper conjured up a vision in the future of fog banks festooned with balloons placarded with the information of their height, and the correct landing directions, and the time when every aerodrome would send up its balloon in the same fashion as fog signals were started at sea. To him that seemed quite practicable. He thought the ground approximator was Mr. Meredith's own invention. Equipped with that, would it be possible to make

landings on any aerodrome in a fog if guided by a balloon which gave the correct height and direction in which to make a landing? He suggested that the members might like to hear the opinion of Flight Lieutenant Oddie, who was the actual pilot in the experiments.

Flight Lieutenant ODDIE: The problem of flying in fog or in cloud had been adequately dealt with by the lecturer, and for the most part he thought they would be in agreement with what he had said, though perhaps not entirely. No doubt to anyone with a mathematical mind, such as Mr. Meredith's, the problem was extremely easy, but its ease or difficulty did not depend entirely upon the instrument with which pilots were supplied. Providing that calm conditions prevailed, and he was not lost in the fog, the instruments at his disposal rendered it quite a simple matter for him to fly in fog or cloud. Unfortunately, however, with due respect to the manufacturers, aeroplanes were not all of the same type; some of them were distinctly more troublesome to fly than others. As the lecturer had pointed out, the problem of flying in fog was not merely one of controlling the aircraft in straight flight; but also of navigating to one's destination. In order to achieve this a perfectly calm state of mind is essential. Although it could be done quite easily on certain aeroplanes that were stable and small, there were other aeroplanes which were large, unstable, and not exactly pleasant to fly, and in which it was difficult to keep a straight course, without taking into question the navigation.

The President had inquired what faith the lecturer had in directional wireless. Speaking for himself, he (Flight Lieutenant Oddie) had very great faith. He had had a fair amount of experience of it, and he thought it was extraordinarily reliable, especially direction finding from the aeroplane. Nobody would pretend that they had gone very far into the problem, but what they had found out was sufficient to show that one could fly safely through a layer of fog on to an aerodrome. If then they might assume that the fog would cease at a certain height and clear air were experienced above it, in which a balloon could be seen, then, with the help of direction finding from the aircraft he thought that a definite problem had been solved. If they had perfectly calm conditions above the fog they could concentrate on navigational flying. Personally, he would have the greatest faith in direction finding from the aircraft to take him to the balloon. The balloon scheme was absolutely accurate for any practical flying. The lecturer had pointed out that one hundred landings had been made with a maximum error of about ninety yards. That could be cut down by half by concentrating on flying close to the balloon, but in view of the fact that they had 500 or 1,000 yards in which to land there was no need for such extreme concentration. If he had to replace the balloon by wireless he would be extremely chary of it, but no doubt somebody else would be performing the test when that happened.

(*Communicated.*)—The principal bone of contention appeared to centre round the bubble as a fog flying instrument. The opinion seemed to be that it was possible to fly through cloud with the assistance of a bubble and compass. It seemed to him that the bubble gives an indication of bank only when the aeroplane is not turning, or conversely an indication of turn when the aeroplane is horizontal laterally.

If a deflection of the bubble occurs, the probability is that not only is the wing down but as a consequence a yaw has also been set up. The pilot, unaided by other instruments is unable to distinguish which of the above two has actually occurred. It is said that the compass supplies this aid. Although it is agreed that for southerly courses this aid can be sufficient, for northerly courses it becomes more difficult as the course approaches north until within certain limits the indication given by the compass is definitely wrong and becomes a danger rather than an aid.

It is thought, therefore, that the compass is quite inadequate as an indication of yaw to the pilot. In practice they found that the application of aileron in cloud only increases the yaw and makes a steady course still more difficult. The method they adopted for cloud flying is to keep the aircraft on a straight course by means of a turn indicator, leave corrections in bank to the inherent stability of the aircraft and use the control column gently in pitch only as indicated by the air speed. If a pitch and yaw indicator is used the problem is considerably simplified. They are, however, of the opinion that the bubble is a useful instrument merely to reassure the pilot that the feeling of slip which he gets sometimes, is in fact pure sideslip and not due to failure of the turn indicator or compass.

Mr. CAPON (Superintendent of Scientific Research, R.A.E.): A year or two ago a demonstration of fog landing was arranged and had to be postponed owing to fog. They were now in a much happier situation, which he thought was in the main due to two things, the balloon system that was suggested by Wing Commander Hill or by Professor Lindemann, and the landing device, for which they were indebted to Mr. Meredith. Those systems had rendered an accurate landing possible in fog. He admitted that it was a short step compared with the distance they had to go, but it was a very important step. He would pay a tribute to Flight Lieutenant Oddie, not only for his courage in conducting experiments with a single engine, but for the skill, enthusiasm and ability which he had displayed in his attention to the problem.

They were all primarily interested in the question of the ultimate prospect of success in landing air transport machines regularly in fog. The situation at present was that if aircraft could be brought to a point near the boundary of the aerodrome, namely, to the point marked in the present method by a balloon, landings could be made successfully. It remained to mark the point by intersecting radio beams or similar methods applicable even when the fog was so deep that the balloon method was not applicable.

He thought that all the indications pointed to the fact that those who were investigating the question of fog landing would be ready before the engine designer was; for an absolute immunity from risk of forced landings in fog away from the aerodrome was evidently necessary.

A short time ago he asked a representative of a Continental air line what he thought of the fog landing position generally. He had only two objections to raise to the general scheme which Mr. Meredith had outlined. One was that he did not think the landing device would operate on the larger types of aeroplane, and the other was that he thought the risk of collision would be very great. As regards his first objection, the early experiments on the landing scheme had been made on a large two-engined aircraft and our only fear had been that it would not operate satisfactorily on a small aeroplane. Mr. Meredith had alluded in his paper to his second objection when he had said that there was no danger if the pilot did not fly in close proximity to the ground. Mr. Capon thought there should be no difficulty in arranging heights for each route, so that the risk of collision would be reduced to a minimum.

He had spoken of Flight Lieutenant Oddie's work, and he would like to express his appreciation of what had been accomplished by Mr. Meredith. The steady progress and the success which had been achieved so rapidly showed the amount of thought and ability which had been devoted to the subject.

Major STEWART (Principal Scientific Officer, R.A.E.): The lecturer had given the opinion that the pitch and bank indicator would be sufficient, and that the turn indicator was redundant. One noticed that Lieutenant Doolittle used the turn indicator. He (Major Stewart) had heard from American sources that he used two, and such a decision would be endorsed by Flight Lieutenant Oddie and by other pilots flying on this side of the Atlantic.

Flight Lieutenant Oddie had laid great stress, and rightly, upon the need for calmness in the mind of the pilot. The quantity of apparatus carried in the Guggenheim experiments called for intense concentration on the part of the pilot in order that the meaning of each instrument indicator may be understood and correlated with the readings of other instruments. The simplicity of the system described by the lecturer should be of great value in attaining the necessary calmness of judgment.

It had to be borne in mind, as the lecturer had pointed out, that fog flying should only be used as an emergency measure. He (Major Stewart) might be taking a pessimistic view, but he did not think that within the next few years people were going to pay money for the privilege of deliberately landing in a fog.

The difficulty with which one was confronted was to find that position in space at which to start the glide into the aerodrome when enveloped by a fog or out of sight of the ground. Mr. Meredith and Flight Lieutenant Oddie had, by their efforts, proved that one could land on an aerodrome once the required starting point of the glide has been established. He (Major Stewart) hoped that in the near future more information, gathered from actual experiments, would be available on the determination of this starting position, so that the pilot might set the controls and engine speed with the certainty that a safe landing on an aerodrome would be made.

Mr. WIMPERIS (Vice-President and Director of Scientific Research, Air Ministry): The method of fog landing so lucidly described by Mr. Meredith was not the first experiment in that direction. Lieutenant Doolittle had been engaged in such work for some time under arrangement with the Guggenheim Fund. Last year, when he was in America, Mr. Wimperis saw Lieutenant Doolittle and his machine, and he came to the conclusion that it was really wonderful that the pilot was able to make use, in the undoubtedly excellent flying he did, of so many instruments. In comparing the instruments used at Farnborough with those used at Mitchel Field it was clear that Farnborough had a very real advantage in the suspended weight system of landing. Flying "into the ground" was bound to be unattractive. Indeed, he noticed in Lieutenant Doolittle's machine a very sturdy net, to catch his head in case the landing was unduly severe. While over in America he (Mr. Wimperis) heard an allusion to the attitude adopted by birds in foggy weather. A pilot rang up the meteorological station to know what the weather was like. The answer he got was: "Visibility nix. Even the birds are walking." So he did not go.

One asked oneself the question, what was going to be the use of this particular work? He felt sure that the balloon method which had been described that night would be of service to the Royal Air Force. But when one came to civil flying, would pilots with valuable machines and still more valuable passengers take the risk? That remained to be seen, but these methods of fog landing were being developed because they had other uses; the information would be published and it would therefore be available for civil aviation. He thought everyone would agree that work of this kind needed to be explored to see what were its possibilities, and it was for those pilots who were flying for civil aviation purposes to make up their minds whether the system was good enough for them; if it were not good enough at the present time, the day might come when by improvements of the radio system the responsibility might be shared between the air and the land. In other words, a pilot would be informed what he ought to do not only by his own instruments but by instruments working on the aerodrome.

Major BRACKLEY (Imperial Airways): There were Imperial Airways' pilots present who had a good deal of experience of flying in fog and cloud flying, but when it came to landing in fog with large commercial aircraft it was another matter.

He could not agree with the lecturer that with a little practice fog flying was easy to the average pilot. His experience was that most pilots found fog and cloud flying most difficult for prolonged periods.

With regard to fog on the London-Paris route, he said that there were a number of days in the year when fog or very low cloud covered the whole of the route.

Regarding the various aids to pilots mentioned by the lecturer, Imperial Airways had tried the R.A.E. " Gyro " rudder control, but they had been unable to get a combination of rudder, elevator and aileron control. They hoped soon to be given the opportunity of trying all three axes.

The pitch and yaw indicator mentioned by the lecturer seemed a very useful instrument and one which they hoped to use in the future.

They had tried the R.A.E. wing coil method of direction finding on their Near East route and had met with considerable success. Pilots reported that they had been able to arrive at their destination with very little difficulty, and they were gaining confidence in that particular method.

He thought they would have to adopt this method of direction finding on their new routes as it did away with the necessity of additional ground equipment.

Mr. F. Tymms (Chief Technical Assistant, Directorate of Civil Aviation): While it was equally easy to be convinced that flying in fog could never be a practicable form of transport as to be convinced that it inevitably must come, there was no doubt that it was incumbent on everyone concerned with air transport to study every aspect of the problem.

The President had suggested in his opening remarks that it was unduly ambitious to wish to fly through fog when all other forms of transport were suspended, but it was necessary to realise that flying, so far from being the most difficult form of transport to carry on in fog, might actually be the easiest, since many of the conditions which are fundamental in surface transport do not necessarily apply to flying. Granted certain conditions, the achievement of which appears to be in sight, then the danger involved in flying in fog will be reduced to the two ends of the flight and principally to one—when the machine is descending. In two-dimensional transport, the danger exists the whole time.

Collision is one of the most serious potential risks in air transport, and it is the more serious at the present time because of the incomplete solution of the fog flying problem, which leads to crowding of the very restricted clear space below clouds. When this solution is complete, as the lecturer pointed out, the risk of collision may be reduced to a negligible quantity by the adoption of height zones. Something on these lines is attempted at Croydon already, in advising pilots of the height at which other aircraft are flying, but the fact that it is impossible to make a pilot fly through the clouds until he can with certainty come out of them safely makes it impossible to apply any such scheme fully under present conditions.

The lecturer had suggested that his aim would be achieved when the system was sufficiently good to be used in emergency. However valuable this result might be, it could not be considered a satisfactory solution for the needs of air transport, whose successful development demands regularity. Further, if pilots were merely to use the system when accidentally caught in fog, they would never attain confidence in it.

Referring to the four systems of radiogoniometry, he said that it was intended to establish at Croydon a directional radio beacon. With the Bellini Tosi system, which had been in use for some years, and the two rotating beacons recently established, there was a choice of three systems on the Continental route. He was glad to learn from Major Brackley that the fourth system which was now being tried on the Egypt-India route, namely, the wing coil system, was proving satisfactory. This system has the merit of reducing the ground organisation costs of an air route.

He noticed that the lecturer had avoided a rather common tendency at the present day to think of the problem of navigation, not only in fog, but generally, as one to be solved by some automatic means which took the whole thing out of the hands of the pilot. He (Mr. Tymms) might be a little old-fashioned, but it always seemed to him that one must leave the responsibility to the pilot. The directive radio beacon and intensive route lighting are two examples of the tendency. Both tend to ignore the possibilities of navigation latent in the pilot and his instruments. Both have the bad feature that their whole powers to aid are concentrated on one line and only by getting back to and following that line can any assistance be obtained from them. With this in mind, the regulations demand certain qualifications in navigation from pilots. In that connection he would like to draw attention to the endeavours being made to get established in this country, a school of air pilotage which would cover the whole of the technical problems of flying, including not only the flying of commercial types of aircraft, but blind flying, cloud and fog flying, navigation and meteorology.

Referring to the wing coil system, Mr. Tymms did not think the lecturer could have meant exactly what he appeared to have said, namely, that the advantage of the system was that it alone supplies all the information required for directional control. It appeared to him that proper navigation by this method involved also the use of the compass.

The lecturer had said that the rotating beacon system gives a correct bearing without ambiguity. Mr. Tymms thought there was an ambiguity, in that, having got a bearing, one had to use other evidence to determine whether to lay off that bearing or its reciprocal.

Mr. WILLCOCKSON: The lecturer seemed to be a little pessimistic about the bubble. He asked why. Most people flying in cloud on air routes found it nearly the most important instrument, as far as night flying was concerned, especially with the new type of machine fitted with the Servo control.

Mr. BRAMSON (Member of Council): Was it not the fact that the bubble was a useful and accurate instrument, providing that by some other indication one could be sure one was not turning while watching it? If they had a machine equipped with a turn indicator, or even a compass (assuming they were not flying blind), and thereby knew they were not turning, surely the bubble would tell them whether they were horizontal or not. On the other hand, if in a turn, the bubble would enable them to make an accurate turn, in so far as the relation between the curvature of their turn and their bank was concerned.

The problem of finding the balloon or its radio equivalent was a problem of finding a point in space which was formed by three co-ordinates or three planes at right angles to each other. One plane was easy to establish—the horizontal one—if they had information as to the barometrical correction for the altimeter, so that they could fly accurately at a given height whilst approaching the aerodrome. The other two planes were difficult to find. One, the vertical plane, passing through the centre of the aerodrome and containing a straight leader cable, could be accurately defined by the latter. The other vertical plane, intersecting the flight path at the point where the pilot ought to throttle down his engine, could be established by an acoustical listening post, or by another leader cable at right angles to the main cable and with a different frequency.

He (Mr. Bramson) did not share Major Stewart's pessimism about what the public would do. The public was given too much credit for technical knowledge of the risks they ran. The public took it for granted that their safety was adequately provided for, just as when travelling by train or other forms of public conveyance they trusted themselves to the responsible persons. What did the ordinary train passenger know of the signalling system? He knew absolutely nothing. He assumed that a perfectly good company to whom he had paid his fare would do the job properly. He (Mr. Bramson) felt practically certain that if a service was run the public would travel by it, whether it flew in fog, sleet,

rain, or anything, knowing nothing about the reasons for their safety but accepting it.

Mr. P. F. Everitt, B.Sc.: He drew Mr. Meredith's attention to a method of navigating aircraft in fog which he had not mentioned. The method was used by the late Squadron Leader Johnston on one of the airships when flying in or above cloud from Cardington to Southampton.

Departure was taken from a known point—the start or some subsequent point—and a course set and maintained; three wireless bearings of some convenient wireless station at three known times were observed; these three bearings at known times determined the track of the aircraft, while one of the bearings combined with the track from the start gave a known fix determining the ground speed made good. Thus the track and ground speed were determined, which immediately, in combination with the course steered, etc., gave the velocity and direction of the wind.

All the elements were thus known, so that a corrected course could be set to bring the aircraft to its destination, the new ground speed, and consequently expected time of arrival being known. The same system permitted of checking positions, tract and wind, whenever a suitable station could be observed.

In order to facilitate the use of this method, Messrs. H. Hughes and Son, Ltd., were constructing a special mechanical computer to give the track from the three bearings and times. Unfortunately it was not possible to show the instrument that evening but, if so desired, that could be done at a later date.

The use of that method did not require any additional instruments, except that the work of the navigator was materially decreased if a suitable mechanical computer was employed.

The method was perfectly sound and had been thoroughly tested and found satisfactory by Squadron Leader Johnston, whose loss was so much to be regretted.

Mr. Olley: He expressed surprise at the lecturer's remarks about the bubble. He (Mr. Olley) had always regarded it as one of the most important instruments, and even without the turn indicator he was quite prepared to take a machine up.

He also expressed a desire to see more experiments carried out with regard to landing in fog, which he was rather dubious about at present. He would not care to take it on himself with any instrument, as he thought the personal element would enter into it too much.

Squadron Leader D. H. de Burgh, A.F.C. (R.A.E.) (*communicated*): He would like to congratulate the author on his perspective of the subject, in which precedence was always given to the operational requirements rather than the technical experiment—an attitude which is noticeably lacking in many technical men.

He thought the author, and also the President, showed undue scepticism about the reliability of wireless.

Under working conditions, such as are met with in the R.A.F., long wave wireless telegraphy failures should amount to less than one per cent. Similarly, long wave D.F. bearings should lie within three per cent of the correct bearing provided the apparatus is properly maintained, and provided the navigator knows the limitations of the system which is being utilised.

But he would like to emphasise that navigation of any sort is *not* an exact science, but rather a summation of various methods of obtaining position lines. No navigator with experience would consider a fix to be a good one, unless it were the result of several observations, preferably employing different methods.

Mr. Meredith made an error in referring to the wireless rotating beacon as having a cardiac diagram in its simplest form. Such a beacon is a rotating loop transmitter, and consequently has a figure of eight polar diagram. There is, therefore, a 180° ambiguity in any bearings taken.

With regard to the subject of the use of the lateral bubble, which became somewhat controversial in the discussion, he, personally, agreed entirely with Mr. Meredith.

His experience of cloud flying, though limited to relatively small aircraft, was enough to convince him that if one has a turn indicator and an air speed indicator—or, preferably, a pitch and azimuth indicator—the aircraft itself looks after its lateral attitude better than he could.

Mr. FRANK RADCLIFFE (of Gloucester Aircraft Company) (*communicated*): Probably one of the most useful and profitable forms of civil aviation will be the Air Mail, and this of its very nature will demand a punctuality and a reliability that is not as yet realisable in the best of present-day air lines. Something approaching 100 per cent efficiency will be requisite for all journeys whether undertaken in summer or winter. The balloon tests at Farnborough and the American " blind flying " tests carried out under the auspices of the Guggenheim Fund have demonstrated that landing in fog is possible under certain controlled conditions, but as yet the next step, viz., the landing of an aeroplane at any suitable spot not fitted with the apparatus mentioned in the paper has not been done.

Clearly the problem is exceedingly difficult and is probably impossible of an unqualified solution. It remains, therefore, to put certain limits on the problem in order to make it reasonably sure of great success in the near future. It had occurred to him that perhaps one solution would be to limit all flying carried out by air mail planes in fog to certain well defined routes which would have emergency landing grounds at regular intervals suitable for the type of aircraft operating on the route in question. This being so, it seems as though there is a good case provided here for the high altitude mail plane, which, in the event of desiring to land in a fog due to some unexpected cause, could then glide from its altitude to such an emergency landing ground, guided by wireless beams as to the route, or located by such beams as the rotating beacon. The value of the altitude would be twofold, it would give the pilot time in which to think and pick up wireless directions, and would also enable the emergency landing grounds to be placed at greater distances from one another.

Flight Lieutenant C. M. VINCENT (R.A.E.) (*communicated*): With reference to the conflicting opinions ranging around the uses of the bubble as an aid to blind flying, Mr. Bramson stated that the bubble indicated a perfect turn. This is quite correct and is therefore considered one of the chief dangers of the bubble as a blind flying instrument. Over a long series of experiments, it has been found that, in nearly all cases of an aeroplane getting into difficulties in cloud, when flying by a compass, A.S.I., and bubble only, the start of the trouble is the same. The pilot feels the machine slipping and everything seems wrong, he looks at his bubble, which gives him an indication. This indication may mean either that a wing has dropped, or that he is turning flat. If he decides to correct by aileron the aircraft yaws badly, due to aileron drag; this yaw, however, combined with the wing being already down produces a perfect turn with the bubble therefore showing central. The aeroplane is not therefore brought back fully to its horizontal position by use of the bubble. The compass can be ruled out for indicating turn owing to its misleading behaviour on northerly courses. If, on the other hand, he applies rudder he notices that the feeling of slip immediately disappears and the bubble becomes central. This therefore is the course usually instinctively adopted in cloud. The perfect turn has now started, the nose of the aircraft drops, the speed goes up, the stick is eased back to decrease the speed, the turn becomes tighter, the nose drops still further; once more the speed goes up, the stick is again eased back making the turn still tighter and so on until the aircraft gets into a tight steep spiral, eventually ending up either in a spin or a structural failure. If, on the other hand, some type of turn indicator is fitted (preferably without a bank indicator attached)

and corrections made only on the rudder and elevators, leaving the inherent lateral stability of the aircraft to look after the bank, this would not happen.

He was, therefore, of the opinion that the bubble as a blind flying instrument is useless. It is, however, very pleasant to have this instrument in the machine from a psychological point of view. He was sure Mr. Meredith would be the last person to suggest that as an aircraft instrument the bubble should be eliminated from the dashboard of an aeroplane. He did, however, after a fair amount of practical experience in blind flying, agree with him that the lateral controls of an aeroplane should be left severely alone when flying in cloud; if not, sooner or later, the machine is certain to get into trouble, of which the consequences may be very serious.

Reply to Discussion

Mr. Meredith said that the chief bone of contention appeared to be the lateral bubble and that he would deal with that point first. It was quite clear that if by any means they could ensure that the motion was unaccelerated then the lateral bubble did give them the measure of tilt and answered the same purpose as the carpenter's level. But he was discussing the question of blind flying equipment. If one could be certain of flying without acceleration one would not need any equipment. It was precisely because one was apt to experience lateral accelerations when flying in cloud that special instruments were needed. His argument was that it was unnecessary to bother about the bubble or to move the ailerons because every aeroplane, thanks to the foresight of the designers, was provided with a dihedral, and if left to itself would come on to a level keel, if the pilot continued flying in the right direction. That had been proved by experiments with automatic controls which had flown aeroplanes for hundreds of miles often in very bumpy weather without using the ailerons at all.

The Chairman had mentioned the sad story of someone trying to work by radio bearings. He would remind the Chairman that in his lecture he had said that wireless was still a tricky ally.

Major Stewart had mentioned the special ability required by the pilot to fly with such a formidable battery of instruments as that used by the Guggenheim Fund. The Guggenheim Fund had something to say on that subject, and he proposed to read an extract from the foreword to their pamphlet, " Equipment Used in Experiments to Solve the Problem of Fog Flying ":—

> " It may appear that the instruments are too complicated and that too much experience is required in their operation. This is not the case, as was actually shown by the Fund's pilot at Mitchel Field. It had been remarked that much of the work done by the Fog Flying Laboratory of the Fund could be accomplished only by an expert pilot. Those in charge immediately countered by saying that a second lieutenant, comparatively unfamiliar with the work, accomplished in a short time most of the essentials required in the kind of flying that had been done by the Fund's pilots. Critics promptly responded that a second lieutenant was continually flying and usually was an expert pilot. Thereupon the Fund's pilot, after a very short indoctrination course, accomplished the same results with a high ranking officer of the Army Air Corps—one who was not as active in flying as the lower commissioned personnel. The conclusions are obvious."

At the same time he must say he agreed with Major Stewart that the battery of instruments was rather formidable, and he thought one should aim at simplifying the flying equipment as far as possible. That was one of his reasons for objecting to worrying the pilot about the bubble, because if he had not to concern himself with the aileron control there was one less instrument for him

to watch. In the fog flying equipment which Flight Lieutenant Oddie used for control of the aeroplane, apart from navigation, he had only employed one instrument, the pitch and yaw indicator, except for occasional checking references to his air speed indicator.

Captain Tymms had said that his statement that a visual direction finding instrument in the aeroplane supplied all the information required for directional control was not quite accurate. What he (Mr. Meredith) had intended to convey was that as regarded maintaining directional *control* of the aeroplane, that one instrument was sufficient. For *navigation* purposes, it was necessary for the pilot to observe the compass and other things. But navigation could be done at leisure, whereas steering had to be done immediately. He wished to stress the distinction between control and navigation.

Major Stewart was afraid that people would not pay for the privilege of flying in fog. Mr. Meredith agreed with Mr. Bramson, who had rather anticipated his reply, that the general public would be only too anxious to throw the responsibility of conveying them in dangerous weather upon the unfortunate and reluctant pilots.

Captain Tymms had expressed a desire for a method which could be used on all occasions, and had disagreed with the division of the problem into escaping from fog and what might be described as wantonly flying in fog. He could not agree with Captain Tymms. Until all the instruments and the aeroplanes had been proved to be very much more reliable than was known to be the case to-day, it would be criminal folly to take passengers up wilfully in a fog if it could be avoided.

Mr. Bramson had mentioned an acoustic system. He thought he had dealt with it in his paper. It was desirable to get an additional check on the position of the aeroplane from the ground, and probably an acoustic system would be the most likely way to do it.

The method of navigation used by the late Squadron-Leader Johnston had been cited. Mr. Meredith was very much interested to hear of it, but he did not think that method conflicted with anything he had said in the lecture, because it was essentially dependent on a radio fix of position. The fact that Squadron Leader Johnston did not use the radio all the time did not affect the issue; he could not do without it. If a pilot wanted to arrive accurately at his destination he must depend upon getting ultimately another radio fix.

He agreed with Major Brackley that most pilots find cloud flying very difficult for prolonged periods with the normal instrument equipment. He suggested that they often make it more so by using the ailerons without a true indication of bank. With regard to his statement that there are a number of days in the year when fog or very low cloud covers the whole of the route from London to Paris, his contention was that the decrease in visibility was not so rapid that a pilot could be enveloped during the comparatively short time occupied by the journey. It must be remembered that statistics of the number of days on which fog occurs are definitely misleading as to the amount of interference with aviation unless one takes account of the duration of the fog on each day. Major Brackley expressed a hope that Messrs. Imperial Airways would soon be given the opportunity of testing a three axes automatic control. He saw no reason why that hope should not be fulfilled, but he doubted whether the extra weight and complication of the aileron control will ultimately be considered justifiable.

While he concurred generally with the views communicated by Mr. Radcliffe, he felt that complete reliability of the engines and equipment should be the ultimate goal rather than multiplication of landing grounds.

He concurred in Flight Lieutenant de Burgh's explanation of the error to which Mr. Tymms drew attention.

PROCEEDINGS

THIRD MEETING, FIRST HALF, 66TH SESSION

The third meeting of the Session 1930-31 of the Royal Aeronautical Society was held in the Lecture Theatre of the Royal Society of Arts, 18, John Street, Adelphi, London, W.C.2, on Thursday, November 13th, 1930, when a paper on "Testing Aeroplane Controls," by Mr. H. L. Stevens, A.F.R.Ae.S., was read. The President of the Society (Mr. C. R. Fairey) was in the chair.

The PRESIDENT: Mr. Stevens brought to the consideration of this subject his long experience at the R.A.E. and as Chief Technical Officer at Martlesham Heath. The paper was very interesting and highly controversial. In his own experience he had never known two pilots to agree on the questions of aeroplane control, and it would be interesting if Mr. Stevens' paper, or the debate upon it, could show that there was any common ground of measurement of aeroplane controls.

TESTING AEROPLANE CONTROLS

BY

H. L. STEVENS, *Associate Fellow*

Introduction

Before beginning this lecture I have first of all to thank the Air Ministry for permission to give it and to explain that any opinions expressed are my own and do not necessarily reflect the opinion of the Air Ministry.

Secondly I must express my appreciation of the honour conferred on me by the Society in asking me to deliver a lecture before them; though I wish they had chosen a less difficult subject.

As regards the title of this lecture it sounds very circumscribed, but you can't test anything without having a fair idea of what its complete function is or without some standard of reference; and in trying to discuss these parts of the problem I am afraid you will find that I have been, if anything, too general.

Characteristics of Aircraft

Aeroplanes, I think, can be divided into three classes. The *obstinate*, which again sub-divides into two, the obstinate and vicious, and the obstinate and self-willed. The *self-controlled* and the *easy going*. There is, of course, no hard and fast line between these groups, but some such classification will help us to fix our ideas.

The *obstinate and vicious* in its worst form is highly unstable, it wants to dive at high speeds and rear at low speeds and the more it gets its way the harder it is to return it to the straight and narrow path. It has a violent turning tendency at slow speeds, and swings taking off and landing. If accidentally stalled it drops a wing and spins straight away, and if allowed to spin for any length of time refuses to come out until the pilot has decided to leave by parachute. Its controls are heavy, more particularly when trying to return it from one of its more violent excursions.

Of course all these things are not likely to happen at once, for if they did the aeroplane would not get as far as a testing station.

The cure for fore and aft instability is to move the C.G. forward. The alternative of fitting a larger tail doesn't always work as the C.G. comes back still farther with the extra weight of the tail. The same applies to a lengthened fuselage if the aeroplane has been originally designed too short, but in my own opinion a reasonably long fuselage is the best insurance against most of the troubles arising from inadequate tail organs.

The cure for a big turning tendency in the air is an increase in airscrew diameter; turning tendency is roughly inversely proportional to the fourth power of the airscrew diameter. Alternatively the fin may be set over, or fin and rudder cambered on one side to divide the tendency between climbing and gliding, remembering that owing to the extra speed over the fin, due to slip stream when climbing, a reduction caused by offsetting the fin will appear only as a turning tendency of about one-third this amount in the gliding condition.

The cure for swinging on the ground is difficult to prescribe as there is very little information as to what it is due to. An adjustment of the position of the wheels relative to the C.G. is sometimes effective.

The cure for dropping a wing at the stall is usually slots. But aircraft without slots do not all drop a wing at the stall, consequently although slots are undoubtedly the most effective cure when the tendency is present it follows that careful design in the first place might have reduced the tendency to small dimensions. Shape and section of wing tips has, I believe, a large effect and a wing section with a flat topped lift curve should be good in this respect. Once the ailerons have been used at the stall, however, I feel that the slot is the only certain preventative of the incipient spin.

The flat spin or prolonged spin from which the aircraft cannot be brought out is a very nasty thing. Here I don't think slots are a cure, in fact I believe in America it was found that opening the slot in a long spin actually made the aircraft spin faster. The cure here is more and better disposed fin area and, above all, a long fuselage. Concentration of the main masses in the longitudinal direction should help according to theory.

The *obstinate and well meaning aircraft* is highly stable. It requires large forces or large control angles or both to break its will. On the other hand, it responds to every gust and can be kept quiet only with difficulty in bumpy weather. It probably also has a large change of trim engine on and off. It knows that the pilot ought to put its nose down when the engine is cut off, and takes care that he does it.

The cures for this case are a backward movement of the C.G., reduction in the areas of the stabilising surfaces, a reduction in dihedral angle and effective balancing of the controls.

The *self-controlled aircraft* may be classed as one with a moderate degree of stability, and can be flown hands off at all speeds. If flown feet off it should only turn gently. It won't spin unless it is made to and comes out by itself if the controls are abandoned. It does not drop a wing at the stall, or if it does, does so mildly; and it can be brought to a level keel with a small loss of height. The nose is well up at the stall, thus giving an effective warning to the pilot. The controls do what one would expect at all speeds and only moderate forces and angles are required for fighting manœuvres. It doesn't respond violently to bumps and any such response is easily checked. This class is really a mild case of the obstinate and well meaning.

Finally we have the *easy going*. This class is stable with controls held fixed, and neutral or slightly unstable with controls free. It has no fixed ideas

about anything and can be put through all manœuvres with small control angles and forces. It has very little change of trim engine on and off. Can be spun and should recover on centralisation of the controls in a not too steep dive. Little response to bumps and easily corrected. It has a small turning circle and is steady in a dive.

Of these four classes it is suggested that class three, the self-controlled, is the class wanted for commercial work and for bombers; while class four, the easy going, is wanted for exhibition flying and fighting.

The above classification is, of course, very arbitrary, and the four classes merge imperceptibly into each other, but it may help to clarify our ideas somewhat, and it shows how stability and control are usually involved together.

Desirable Characteristics of Controls

Having now taken a very general survey of aircraft as they exist, let us now get down to details and consider what we might ask of controls if we had Aladdin's lamp and it worked.

Control Instinctive

First and foremost I think control should be absolutely instinctive. It is true that anything becomes instinctive after sufficient experience, but in a game like flying, where a mistake may be at the least very expensive, the fewer new instincts one has to acquire the better.

In the ideal case one would merely sit in the aircraft and move one's body slightly the way one wanted to go and the aeroplane would duly go there, quickly or slowly, according to the speed of one's own movements.

This being perhaps slightly futuristic, the next best thing is a lever or levers which we move in the direction we want the aircraft to move. And now I must confess to a dreadful heresy; I think the rudder controls work backwards and violate the rule above for an instinctive control. All the other controls that I can think of fit in. Fore and aft and laterally you move the control stick about its axes in the same direction as you want the aircraft to move about its corresponding axes. You push the throttle forward when you want to go forward and pull it back when you want to stop. You pull the brake lever back when you want to stop. Even the switch is right, the knob is up for going up and down for coming down. The tail wheel works the same way as the stick. In the case of the rudder bar, when you want the right side of the aeroplane to go on and the left side to slow up, you push the left side of the bar forward and the right side back. If you imagine an aeroplane controlled by a wheel on top of the stick in which fore and aft and lateral movements were as at present, but the wheel worked the rudder, you can see how absurd it would be to connect the wheel up the opposite way to a car wheel. I believe it is a legacy from the sea on which, I understand, you port your helm when you want to go to starboard, and the rudder bar being tucked away where we can't see it we have got used to it and it has become instinctive.

Nevertheless it causes some trouble to beginners.

Control Progressive

The next ideal condition is that the response of the aircraft should be proportional to the control movement in all conditions of flight and should be correspondingly rapid as the forward speed of the aircraft is increased. In driving a car on a perfectly non-skidding surface a given angular displacement of the wheel produces an angular velocity proportional to this displacement and

to the speed of the car. Now if it wasn't for gravity I think you would find that a given fore and aft movement of the stick in an aeroplane would also produce a turn in the appropriate plane of a given radius, and a given lateral displacement a corkscrew motion of a given pitch, both independent of the speed of the aircraft, consequently as in the case of the car the angular velocity would be proportional to control angles and forward speed. I haven't thought whether the analogy can be driven further to include the rudder, and as, in practice, gravity cannot be neglected and these ideal conditions do not apply, perhaps it is not worth while.

In the practical case, the wings which supply the main portion of the centrifugal force for turns and loops also support the aircraft. The extra lift for centrifugal force is supplied by temporarily increasing the incidence. As the speed is reduced the weight can only be supported by increasing the incidence also. Beyond a certain incidence the lift ceases to increase. Consequently, as the speed is reduced, the margin of lift for centrifugal force gets steadily less until at the stall no margin is left. This is why the result of gliding into land too slowly is usually a "pancake," as there is no excess lift available to curve the path of the aircraft parallel to the ground, or in ordinary words, flatten it out, so the ground does it instead.

In other words and returning to the car analogy, if the car behaved like an aeroplane and you drove up to a curb too slowly, you wouldn't be able to turn and skirt it but would slide into it bodily sideways. Speed is a very important factor, not only in the rate of rotation obtained from a control, but also in the radius of turn obtained, particularly in the vertical plane as required for flattening out to land.

However, at speeds reasonably removed from the stall, it appears fairly easy to achieve a control which will give a response proportional to its displacement. That implies no dead spots, no sluggishness for small control angles and then a sudden response for slightly larger angles. And the aircraft should be capable of being returned to an even keel as readily as it can be disturbed from it.

Controls Independent

The next ideal condition is that the controls should be independent. By this I mean that movement of the controlling organ should produce initially at any rate, a motion of the aircraft about the same axis and that axis only. This is satisfied by the elevator control and fairly well by the rudder and aileron controls at all normal speeds. It is very far from satisfied by normal aileron controls near the stall. I will return to the problem of the stall later, but this defection of aileron controls at the stall and the pronounced rudder or yawing effect which they give is all the more dangerous because it is different from their effect in normal flight. In the ideal case all the controls would have similar relative effects at all speeds down to the stall, where, of course, the longitudinal characteristics must change, but the ailerons and rudder should continue to work for some time after this.

Controls in Harmony

Next the three controls should be in harmony with each other. By this I mean that the angles and forces required from each for general manœuvring should be of the same order. I don't know whether this can be specified more precisely; as a tentative suggestion we might lay down that equal forces and angles on the pilots' three controlling units should produce equal initial responses about the three corresponding axes. This suggestion would, I think, lead to relatively lighter ailerons than we have at present. We might perhaps go even further with the ailerons and make them lighter than the others as the direction

in which one moves the control to operate them is the one in which it is hardest to exert much force. The objection can be raised that in practice one does not want to rotate at an equal rate about all three axes simultaneously, consequently, if the above specifications were satisfied equal forces would not be simultaneously experienced on all the controls; on the other hand, it seems reasonable to have to exert the biggest force about the axis one wants to turn round fastest.

This harmony of controls is more important than is, I think, generally realised; besides making all the difference between a really pleasant and an indifferent aeroplane, it may have quite a big influence on safety. There is no more dangerous aeroplane to the beginner than one in which light elevator and aileron controls are combined with a heavy rudder and a pronounced turning tendency.

General

Response must be quick and snappy, especially for fighting and particularly for changing from a turn of one hand to one of the other. This manœuvre shows up heavy and ineffective ailerons quicker than any other.

The controls must be light to operate. I suggest a maximum force of ten pounds for any reasonable manœuvre. And friction should not be tolerated, except perhaps in the rudder control where it is not particularly important and can even be beneficial in stopping hunting and in holding the rudder against a slight turning tendency. Even then it goes against the grain and other methods should be used if possible.

The controls must not be tiring on long flights. This implies a suitable amount of balance and not too much lateral stability or too much work will be required correcting bumps. Also there must be very little turning tendency. A continuous pressure required from one foot of only three pounds is tiring after an hour's flight.

For getting off there should be no tendency to swing, the tail should come up quickly and the attitude should be well under control.

For landing there must be enough fore and aft control to make a three point landing neatly. The aircraft must not drop a wing at the last moment and again it must not swing when speed is lost.

For both landing and taking off the change of trim, due to slipstream, should be small, otherwise there is danger of a steep dive following engine failure, taking off and vice-versa, an attempt to save a bad landing by opening up the engine may result in a bad stall.

Control at the Stall

Returning to control at the stall, I must go very briefly over ground that has been covered before. Why does an aeroplane become an altogether different thing laterally when flown too slow? Referring to the ordinary lift and drag curves (Fig. 1), the lift curve is seen to consist of an almost straight rising portion, a fairly sharp maximum followed by a portion of reversed slope. The drag curve starts horizontally and continues with a slope getting steeper as the incidence increases. Now imagine the aeroplane rolling so that the port wing is going down and the starboard wing rising. The incidence of the port wing is relatively increased as it is falling towards the air flow and that of the starboard is reduced as it is rising away from the flow. Below the stall this gives an increased lift on the port side and a decreased lift on the starboard side, tending to stop the motion. There is also an increase of drag on the port side and a decrease to starboard which tends to turn the aeroplane towards the lower wing. But above the stall the lift effect is reversed and the drag effect largely increased, consequently the roll tends to increase and the turning effect is also increased, resulting in a violent rolling turn or the beginning of a spin.

Now consider the effect of ailerons at the stall, first, without any rolling action (Fig. 2). Small downward movements are seen to give a small increase of lift, but large downwards movements may even lead to a loss of lift. Upward movements always give a reduction in lift. Downward movements give a large increase in drag and upward movements a large reduction. Consequently, although the initial response may be in the right direction it is immediately overcome by the roll due to the turn and the aeroplane ends by rolling against the ailerons.

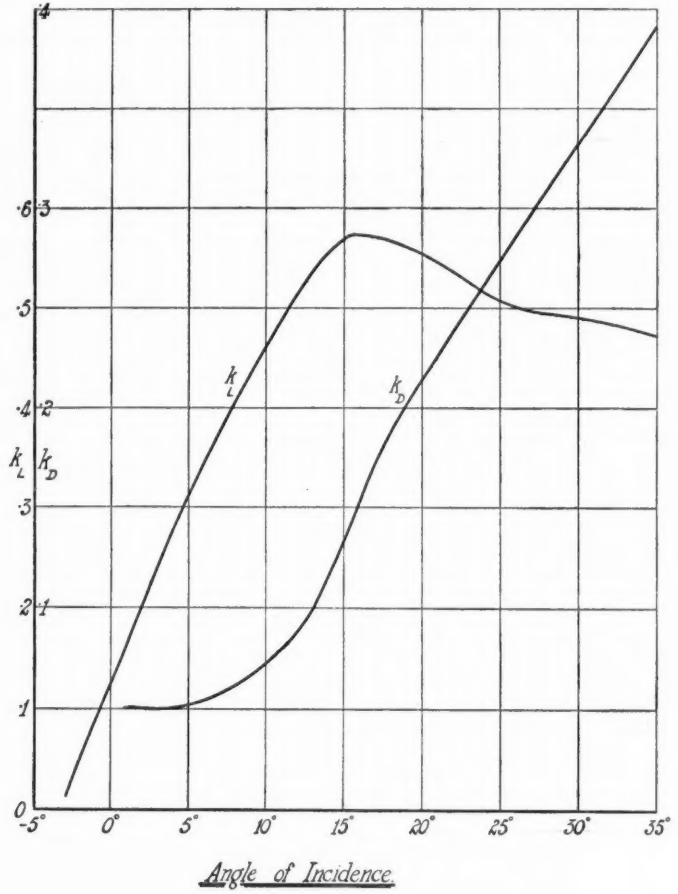


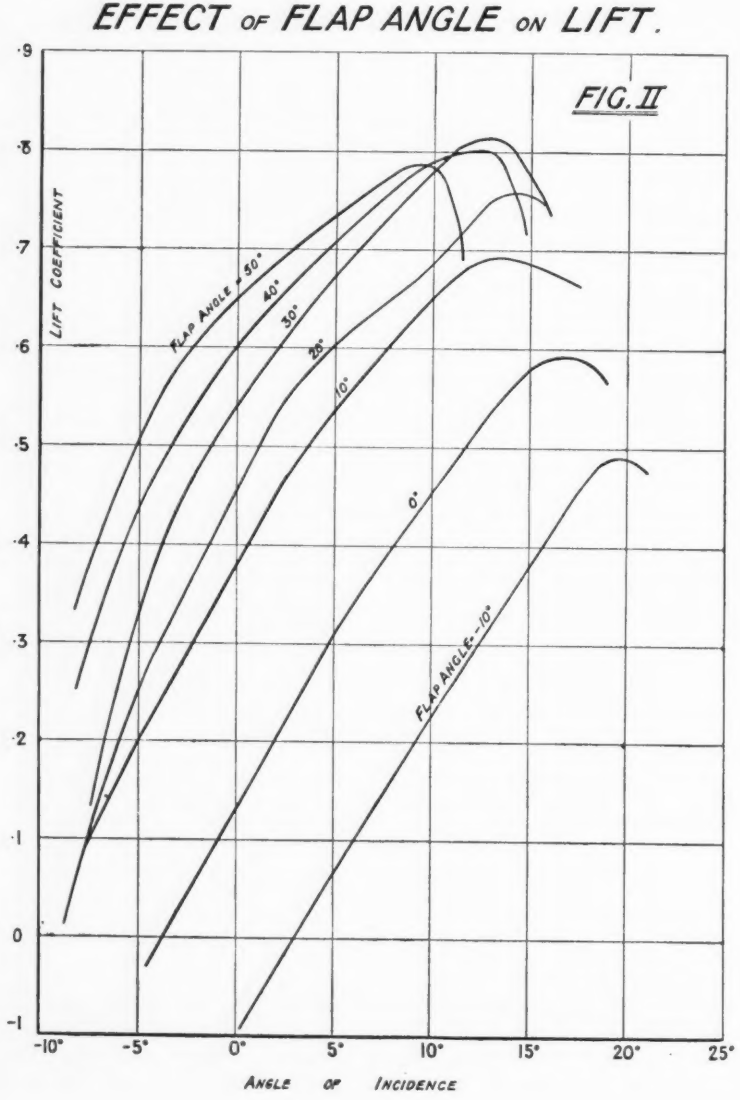
FIG. 1.
Avro 504K. Model Lift and Drag Curves.

If the ailerons are used against a roll already developed the result is worse still as larger incidences have already been reached on the wing it is desired to raise.

One cure for this state of affairs that suggests itself is to produce a wing whose lift curve, instead of dropping after the stall, runs along horizontally.

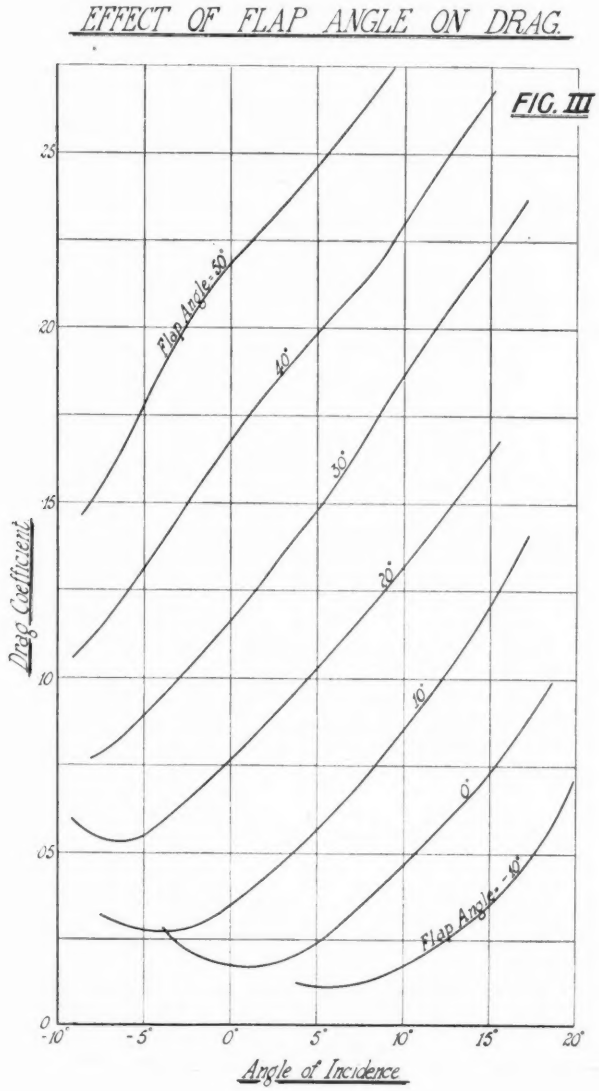
This method has been used with varying effect. It is possible to get lateral stability this way, but to get control is much more difficult as this flat top property is a characteristic of low lift sections and as soon as the ailerons are used one side becomes a relatively high lift section and the flat top property is likely to be lost.

The obvious cure is to arrange that the wing tips do not stall until long after the rest of the aeroplane. The well known slot in its various forms is an example



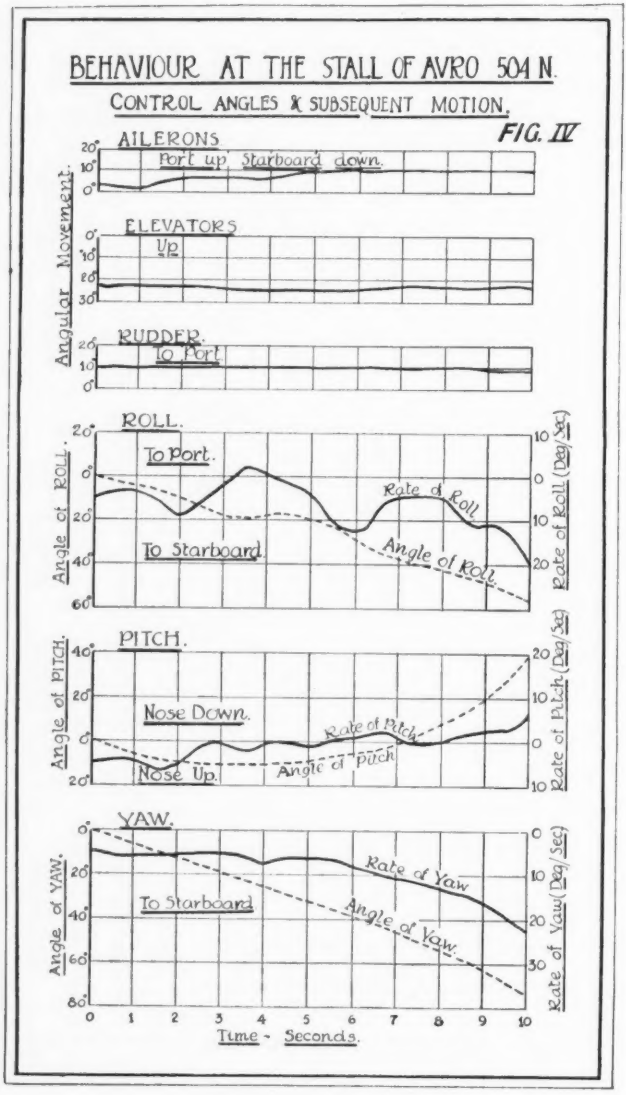
of this. Washed out wing tips and tips of different section from the rest of the plane are other ideas. Again the ailerons may be used differentially, the up going one moving much more than the down; an extreme case of this is control by moving the ailerons up only. Floating ailerons or wing tips can be used; this is a kind of variable wash out. Wing tip shape may be important. I think the trailing edge should always be shorter than the leading edge.

All these devices, except the slot, lead to loss of lift which is not wanted when landing. If the slot is pushed to its logical conclusion, the aircraft becomes



slotted all over and then some extra device is wanted at the tips and we have the intercepter. This again is in the class where control is obtained at the expense of lift and by the time we have slots cum ailerons cum intercepters the aeroplane is getting rather a Christmas tree.

One other method that has been used is to reduce the longitudinal control until the aeroplane cannot quite be stalled without a yank, at the same using a wing section of low drag at the stall. This results in the attitude near the stall



being very much nose up. Consequently the pilot gets a very good warning of impending stall and is also able to make a three-point landing without actually stalling but at a speed very little in excess of the stalling speed.

Strange forms of lateral control have also been tried, but they usually seem to result in a snatchy non-progressive result.

Spinning

Two forms of spin are indulged in, the short and the prolonged.

The short spin is one of two or three turns, it is often used first one way and then the other to relieve the tedium of a long descent. The flick stalled turn used in fighting is really a case of the short spin. The short spin may also be got into accidentally in clouds. Recovery should take not more than half to one turn and preferably should occur with controls abandoned and the aeroplane given its head.

The prolonged spin is one of from six to eight turns. It is of doubtful value, but it may be got into accidentally. Aircraft likely to do it are put through this test at Martlesham to make sure they are O.K. before going to service. Recovery should take place in not more than three turns, controls may be reversed, but preliminary centralisation must not lead to a spin from which subsequent reversal takes a long time to stop the spin. Such a condition has been found on one or two occasions.

The cure for difficulty in getting out of a spin is increased or better disposed fin area. Deepening the fuselage at the rear and leading it to a vertical knife edge has a similar effect. A good long fuselage to start with is a valuable insurance against this trouble.

Diving

The areoplane must be a steady gun platform on a dive and must not require large forces to dive it. This involves roughly neutral stability at high speeds and accurate lateral trim which in turn involves wings stiff in torsion.

If the aeroplane is unstable fore and aft, it will hunt, or the forces to pull out will increase so rapidly that recovery is difficult.

If it is out of lateral trim, it will develop a corkscrew motion in the dive that cannot be controlled. Careful design of rudder balance is also necessary to prevent directional hunting.

Testing

Now here we enter on a regular battle ground, at any rate as far as Martlesham Heath is concerned. At present our reports on general handling are based on opinions only, although admittedly expert opinions. They are not backed up by any measurements to show how good or how bad an aeroplane is. Words such as heavy, sloppy, spongy appear instead of nice concise figures such as the minimum turning circle is of 100 yards diameter and can be encompassed at 140 m.p.h. using a force of 7.5lbs. The aeroplane can be brought from a 60° bank to port to a 60° bank to starboard in 4.5 secs. at a speed of 150 m.p.h. with a force of 15lbs. Also we are told that our opinions change. Well, I hope they do. If aeroplanes did not improve and standards become higher, it would be a poor outlook for the aeronautical industry.

When we have several aeroplanes built to the same specification in competition I think Martlesham can be relied on to say which is the best on controls, and even if we had figures such as I have suggested above, or could allot a system of marks for control features we should still be in a difficulty for the best aeroplane on controls may not be the best in performance. How are we to fix the relative value of the two? It will still be a matter of opinion whether

100 marks for control and 90 for performance is better than 90 for control and 100 for performance.

Our main job is to assist the Air Ministry to get the best aircraft into the Service and to get them there quickly. If two aircraft out of a bunch are both good, each excelling the other in some features, they will probably both go to service trials, and the final decision may depend on such things as robustness of construction or ease of maintenance.

I have an idea that it might assist the aircraft industry generally if the contractors' pilots were all allowed to fly the winner of a competition. They would then have a better idea of the standard aimed at.

There are three distinct points of view about this question of measurement.

(1) From the Air Ministry point of view, if they are satisfied that they are getting the best aircraft without figures, they won't want us to spend extra time getting figures.

(2) From the contractor's point of view, figures are desirable as giving them a standard to work to.

(3) From the research point of view, figures are essential, but Martlesham is not necessarily the place to get them.

Everybody would like figures if they could have them *without holding up tests*. It must be remembered that Martlesham forwards about 400 reports a year. Admittedly, many of them are very short, some just single pages and a photograph, but even those take time in examination, etc. to prepare; hence any increase in the work per aircraft must have big advantages if it is to be adopted. Extra staff is no solution, as you can only do one test at a time on each aircraft.

However, let us assume that we are going to give definite measurements; what can we measure? The things to be measured must be such that a large or small value of them definitely means a good aircraft. It is no good sending up a lot of figures if the winner on points is not the best aircraft after all.

Please do not think I am decrying the value of figures; I realise as much as anyone the difficulties of a system under which an aircraft which has cost a lot of money to build and is somebody's child, is turned down on what the father may think the inadequate opinion unsupported by facts of a few pilots.

So we have been looking round for things to measure and means to measure them, bearing in mind that the things we measure must not involve extra flying and the means to measure them must be fitted in five minutes.

Consider the problem under three heads.

(1) Control at the stall; this includes spinning.

(2) Fighting manœuvres.

(3) Control in normal flight.

Improvement in (1) means safety from vicious aircraft.

Improvement in (2) means safety from vicious enemies.

Improvement in (3) is a matter of comfort only and therefore is relatively of less importance. Hence, as there is only a limited amount of time and money available, attention should be concentrated on (1), and (2) and (3) should be left to the contractors themselves.

Control at the Stall

Here we are looking for the danger of starting an accidental spin and for any difficulty in recovery from short or prolonged intentional spins.

Tests Tried

(1) *Tyros Take-off.*—This test was used with some success in a competition for training aircraft. The aeroplane is flown on a steady climb, the engine is

cut off and the aeroplane turned with the nose still well up, the idea, of course, being to simulate the effect of engine failure taking off, followed by an attempt to return to the aerodrome. This test yields no figures but gives some valuable facts about the aircraft.

(2) *Stalled Glide*.—The aeroplane is held as steadily as possible for one minute with the tail setting at the negative limit, stick right back, engine off. A recording A.S.I. is carried and the fluctuations in the record give some idea of controllability in the stalled state. Here we have actual figures of some value.

(3) *Sustained Spins*.—The C. of G. of the aircraft is adjusted to the back limit for the final test in this series. The tail is set right back, the engine cut off and at least eight turns of a spin carried out. The pilot then endeavours to recover, first by centralising the controls and then by reversing them. The important figure obtained in this test is the number of turns required to recover. We also obtain the height drop in the spin and during the recovery and the time for each turn. A recording accelerometer is also carried and its record may be of considerable value in the case of difficulty in recovery.

Fighting Manœuvres

At the moment we have here opinions only, though we carry accelerometers on these tests also and some use may be made of the records. Mock flights are sometimes carried out, interchanging pilots. The value of control at the stall and also of low loading is shown up in these tests.

It is possible that in the near future we may be able to give actual figures for the smallest turning circle, the minimum time to complete a figure of eight and the forces that have to be exerted at different speeds.

Equipment Available

There is any amount of equipment available in the way of measuring instruments, but most of them take a considerable amount of work and time to fit.

Force-Measuring Devices

(1) *Force Measuring Stick*.—This takes the form of a spring knob that can be attached to the stick. It reads directly the force exerted in one direction. It is easy to fit, but something giving a continuous record would be better, hence we have:—

(2) *Force Recording Stick*.—This gives a continuous record of forces in both directions, but is very much harder to fit, especially in the more modern aircraft whose cockpits are already sufficiently crowded.

(3) *Force Recording Rudder Bar*.—Gives a continuous record of the force on the rudder bar and is even harder to fit.

(4) *Control Wire Force Indicators and Recorders*.—These have been developed to provide something which could be inserted in a control wire, taking the place of the ordinary turnbuckle and therefore being easy to fit. They are of two forms, in the simplest an indicator is moved and stays put at the maximum force recorded, the other form is a recorder. So far they have not been successful. The simple form is difficult to interpret and the take-off and landing forces are often found to be greater than those obtained in the air. The recording form at present has too much friction. Some line such as this seems, however, to be the correct one if records of forces are to be obtained by testing stations.

Devices for Measuring the Flight Path

(1) *Accelerometer*.—This instrument takes a continuous record of the air force acting on the aeroplane, perpendicular to a given plane and therefore of

the accelerations in the same direction. This is easy to fit, often used, and provides valuable data.

(2) *Rate of Turn Recorder.*—This is a very complicated apparatus. It consists of a windmill-driven generator supplying current to three gyros with axes mutually at right angles. The precessional forces of these gyros compress springs and provide a record of the rates of turn about all three axes. Essentially a research instrument and a very valuable one. Has been used in investigations on spinning and control at low speeds.

(3) *Camera Obscura.*—This enables a ground observer to plot the path in space of an aircraft. Not much used at present.

(4) *Cinema Cameras.*—These have been used in various forms. Operating from the ground and photographing the aircraft. Carried by another aircraft and photographing a following aircraft. Carried on the aircraft itself and photographing the sun, the horizon, or the ground.

Devices for Measuring Control Angles

(1) *Simple Measuring Devices.*—These have taken the form of pointers in the cockpit, or in the case of ailerons, out on the wings which move in unison with the control surfaces. These are intended for steady flight rather than for manœuvring.

(2) *Recorders.*—Wind or electrical driven devices which can be fixed on wing, tail plane, or fin, and take a continuous record of the movements of the control surface. These are not easy to fit, though one feels they ought to be. They have been much used for research, particularly on control at low speeds.

Synchronising Devices

These are necessary to enable one to correlate the records from the instruments mentioned above. As these instruments are usually optically recording by means of electric light, the synchronising device takes the form of a windmill-driven clock that interrupts the records at regular time intervals.

This list will show that there is no lack of equipment and most of it is in daily use at the research establishments. A set of figures from one of the reports on control at low speeds will show the uses to which it can be put. Nearly all of it, however, suffers from the disadvantage that it takes a long time to instal, cannot therefore be transferred quickly from aircraft to aircraft, and requires the services of an expert to look after it. Hence we must have something simpler for testing stations.

I think the maximum amount of information we can expect to get in the course of type trials is as follows:—

(1) General opinion of three or four pilots used to the particular class under test.

(2) Accelerometer records in handling and fighting trials, also in spins in which, in addition, the turns and height drop both in the spin and to recover will be observed.

(3) Actual measurements with a camera obscura and maximum control wire force indicators of some manœuvres such as a figure of eight, giving the time to perform the manœuvre and the maximum force required.

Lessons from Experience

This lecture would be incomplete without an endeavour to record any lessons one has learnt from testing controls and analysing the reports of others.

(1) Controls must be harmonised.

(2) Friction should not be tolerated, except perhaps in the rudder controls, and not there if it can be avoided by other means.

(3) Small chord controls down to 30 per cent. are adequate for ailerons, 35 per cent. for elevators and 40 per cent. for rudders.

(4) It is better to have ailerons on all four planes.

(5) A long fuselage is an insurance against many troubles. The American rule that the tail organs should be outside the circle containing the wings is a very sound one.

(6) Fins and rudders must not be blanked by the fuselage at large angles of incidence.

(7) With ailerons balanced by the set back hinges, 35 per cent. balance can be used if the nose is symmetrical and 25 per cent. if it is of the Frise shape. Rigging down the ailerons makes the Frise type heavier and the symmetrical type lighter.

(8) A plain open slot gives lateral stability at the stall. A slot coupled to the aileron and working from the closed position gives control at the stall, but not stability. The latter arrangement working from the open position should give stability and control, but has been disappointing in practice.

(9) The slot-cum-aileron-cum-interceptor does almost all one wants, but is getting too complicated. We want to evolve something new altogether, but it must work smoothly.

(10) Some aircraft without slots do not drop a wing at the stall and this property seems to be a function of wing section and tip shape. The flat-topped lift curve produces this effect on some aircraft, but not on others.

(11) Slots are not a cure for the flat spin.

Conclusion

To sum up, there are three kinds of testing:—

(1) Research.
(2) Competition.
(3) Development.

Research on controls is, I think, largely developed to the end of finding out how nearly model and full-scale aeroplane agree. Given all the properties of a model the expert mathematician can predict what it will do under all circumstances and what forces are required to make it do anything. All the properties, however, involve such an enormous mass of measurements that research also has to be directed to finding out how few are essential and how many we can do without.

Competition.—When we have several aircraft to the same specification in competition, we merely have to choose the best. This, I think, can safely be left to the opinion of expert pilots.

Development.—Here we are concerned with how to improve aircraft. In a competition we find the best and in order to perpetuate its properties in other types we must find out why it is the best. To do this we must find out what pilots do with it, what forces they use and how it responds. We must note in what respects the best aircraft differs from those that are not so good and then we go back to research to find out how to make the best better still.

Finally, I must again call attention to the fact that any opinions I have expressed are entirely my own. In such a controversial subject I can hardly expect that they will be generally concurred in, but I hope they will result in others with more experience coming forward to join in the discussion.

Discussion

The President: Mr. Stevens appeared to consider that there were two ways in which aeroplane controls should be tested. The first was by the use of somewhat elaborate apparatus by which the forces and the results of the application of those forces could be measured, and the second was by considering the

opinions, severally and jointly, of selected and experienced pilots. Apparently the measuring apparatus was very complicated and difficult to use, and the tests would occupy a considerable time. On the other hand, an objection to relying upon the opinions of pilots was the tendency on the part of most pilots to settle down to a definite type of control, so that they might not look with favour upon types of control to which they were not accustomed. When a new machine came into service it was usually objected to strongly from the point of view of control. Then the pilots gradually became accustomed to it, and after a year or two it became the standard for that class of machine, and anything new thereafter—whether heavier or lighter, better or worse—was not popular. That criticism did not necessarily apply to the expert test pilots at Martlesham. At the same time, those pilots, by reason of the fact that they were accustomed to high standards, might demand too much from any type of machine made for general purposes. Commenting upon Mr. Stevens' reference to the timing of a figure of 8, the President asked if he could elaborate that suggestion and produce a schedule of tests involving simple operations, such as the timing of a figure of 8 or measuring the distance from one bank to another, which would provide a definite check on the otherwise unchecked opinions of pilots and so give the results required without the necessity for the use of elaborate research apparatus.

Squadron Leader T. ENGLAND (Messrs. Handley Page): He regarded the classification of aeroplanes into three categories—the obstinate, the self-controlled and the easy going—as being a very clear and a very clever method of classification, for it was a very easy matter for a pilot to say in which of those categories a machine should be included. He agreed with Mr. Stevens that a machine with a long fuselage was better than one with a short fuselage, not only in the matter of stability but in regard to ease of handling generally. Many designers with whom he had discussed this matter had held that a machine with a short fuselage was just as good as one with a long fuselage, provided the tail was in the right place, but, unfortunately, his own experience did not support that view. With regard to the statement in the paper that the cure for dropping a wing at the stall was usually slots, but that aircraft without slots did not all drop a wing at the stall, he suggested that in making that statement Mr. Stevens had, unwittingly, been somewhat unjust to the slot, since it was common knowledge that aircraft such as Mr. Stevens had described, which could be stalled, were invariably vicious in the case of the stall turn. He hoped Mr. Stevens would give some enlightenment on that matter, because a good deal of misunderstanding arose in discussions concerning the stall; the stall and the stall turn were somewhat different from one another. As to the flat spin, very exhaustive tests and researches had been carried out in that connection by Dr. Lachmann (who was associated with Messrs. Handley Page), who had prepared a written contribution which would be forwarded to the Society.

The statement made by Mr. Stevens with regard to the rudder was quite right; there was no doubt that the rudder did work backwards, and Squadron Leader England considered that the beginner would experience no greater difficulty with the rudder than at present if it were reversed. After all, if one were riding a bicycle and wished to turn to the left, one did not push the left hand forward. Discussing throttle control, he agreed that in British aircraft the throttle control was pushed forward when it was desired to rise; in certain types of foreign aircraft, however, the throttle worked in the reverse direction, and he suggested that if they could arrive at a standard method of fixing controls it would be better for all concerned. He was glad that Mr. Stevens had dealt exhaustively with the problem of control at the stall. There was a system in vogue in certain aircraft by which longitudinal control was reduced in order to avoid loss of lateral control. He regarded it as being undesirable to be unable to stall an aeroplane. Although, by being unable to stall an aeroplane, it might

not lose lateral stability, so that there was an advantage in that respect, another complication arose, *i.e.* that of landing in restricted areas at flying speed or, alternatively, landing in a high wind. Either of these manœuvres was very difficult in cross-country flying in an aircraft which could not be stalled, and in his opinion an aeroplane which could not be stalled was just as dangerous as one which could be stalled and which was devoid of lateral stability.

It was very gratifying to note that Mr. Stevens was in favour of contractors' pilots being allowed to fly the winners of competitions. The adoption of that practice would be advantageous; it certainly would enable the pilots in question to obtain a very good idea of the standards at which Martlesham and Felixstowe were aiming.

Judging from the remarks made in the paper concerning the interceptor, obviously Mr. Stevens was not acquainted with the latest research in this connection. Messrs. Handley Page had done a lot of work in connection with the interceptor; he would not deny that at first it had been complicated—and most inventions, unfortunately, were at first " gadgety "—but the interceptor had emerged from the experimental stage, and not only did it work smoothly but it was devoid of gadgets and complications. The latest device—which he hoped that Martlesham and the R.A.E. would soon have an opportunity of testing—was extremely simple, and gave all the control that was needed at the stall, according to present-day standards.

Squadron Leader BENNETT-BAGGS: He agreed with Mr. Stevens concerning most of the points raised in the paper. With regard to instruments for measuring controls, however, if one were going to have instruments for measuring all the controls and results which adduced 1lb. on the rudder, 1lb. on the lateral movement and 1lb. on the fore and aft movement of the stick, who was to decide which was correct? Naturally, an experienced pilot would have to be consulted as to whether 1 or 3lbs. was enough or too much, for each type of aircraft. Harmonised control would be, to some people, ½lb. on all three controls, whereas another person would say that a little more on the rudder and less on the stick would be considered harmonising. In view of the complications of fitting of these instruments and the questionable satisfaction of the results, he would still suggest the practical experienced pilot to determine this question. With regard to the rudder bar, he recalled a flight on which he was accompanied by a " large " lady, who had occupied the front seat and had sat well over to the right when he had banked and ruddered to the left. It had occurred to him that this was wrong. If he were trying to turn to the left it would be upsetting his balance entirely to push on the right foot; he had no strength there, because all the weight of his body would appear to be on the left.

Finally, he expressed strong concurrence with Mr. Stevens' opinion that contractors' pilots should fly all types of winning aircraft, for they could learn a lot by so doing.

Squadron Leader HAIG: He recalled that about five years ago Mr. Stevens and himself had been asked to evolve a formula for " manœuvrability," a formula had been evolved and a standard aircraft had been used to try it out, but it was found that it did none of the things it was supposed to do. He asked whether any further work on this had been done.

Commenting on the references made to the use of a long fuselage, and to the use of slots, interceptors, and so on, he uttered a warning that the fitting of all these various gadgets added useless weight to aircraft, which were already too heavy and rarely paid their way. There was a distinct possibility that in striving for good control a lot might be lost in the way of performance.

The instruments Mr. Stevens had mentioned seemed to be exceedingly accurate and difficult to fit, it might therefore be worth while to have two sets of instruments, one set being only moderately accurate but easy to fit so that

these could be used normally, and it would then only be necessary to use the others for the purpose of checking special queries.

Finally, Mr. Stevens had not mentioned the tail trimming device fitted to many machines. In some of these it was exceedingly awkward to use, and when testing suggested that account should be taken of the rapidity and ease of working of this device.

Mr. H. B. IRVING (N.P.L.): Mr. Stevens personally had been very largely responsible for the design and development of most of the instruments he had described. Discussing the statement, under the heading " Lessons from Experience," that fins and rudders must not be blanked by the fuselage at large angles of incidence, he agreed that this was very important, but that its importance arose chiefly in connection with low speed control. When considering spinning it was very important that the fin and rudder should not be blanked by the *tail plane*. In model experiments it was found that, as soon as the rolling motion of a spin was started, the blanking by the body disappeared almost entirely, and that a new blanking came into force, viz., the blanking, by the tail plane, of fin and rudder. In a flat spin sideslip of the order of 45 degrees might occur at the tail, and under these conditions the tail plane had a most remarkable effect on fin and rudder in most aeroplanes; there had been cases in which the effect was not only a blanking, but rather a back-wash, by means of which the force on fin and rudder was actually reversed, so that instead of having a force of 10 damping the motion there might be a force of -5, *i.e.*, a force helping the rotation in the spin. The remedy for that sort of thing was the deepening of the body and the raising of the tail plane so as to give as much fin and rudder area as possible below the tail plane. As to the statement that slots were not a cure for the flat spin, he said that the indications from model experiments were that the effectiveness of slots in the spin decreased as the angle of incidence above the stall increased. The effect of slots at and just above the stall was great, but it was reduced as the angle of incidence increased—and it must be remembered that in the flat spin the incidence might go up to anything from 45 to 60 or 70 degrees.

One of Mr. Stevens' conclusions was that research on controls was directed largely to finding out how nearly the results obtained with model and full-scale aeroplanes agreed. Mr. Irving believed, however, that the aim of research should be much wider than that. We should like to be able ultimately to predict from model experiments—from simple model experiments, if possible—in what category an aeroplane could be placed as regards its controllability from the pilot's point of view. So far we could not do that, even if we had all the model results complete, because we did not know what sort of manœuvres on which to make calculations with our model results. It was neecssary first that full-scale experimenters should determine the manœuvres which gave reliable criteria of controllability.

Flight Lieutenant CAHILL: Discussing the control lay-out, particularly with regard to the rudder bar, he thought that in a steep turn the present lay-out seemed normal. Under present conditions the pilot put his foot up to get the nose up, and it would be more difficult to put the foot down in order to get the nose up.

Mr. J. S. BUCHANAN (Assistant Director Aircraft, Air Ministry): Like Mr. Irving, he quarrelled with Mr. Stevens' conclusion as to the aim of research. The object of all the experimental work on controls, he urged, was to determine the best type of control to fit to aircraft, which would need the minimum amount of operation by the pilot, and further, to ensure that they did not fit controls which were likely to be dangerous to an aircraft. It seemed to him that if they relied only upon the opinions of pilots—who inevitably preferred the light control —they might in many cases be led to adopt controls which were in many respects dangerous in use. Commenting on Mr. Stevens' remarks concerning competi-

tive aircraft at Martlesham Heath, he said it was an admirable thing that there should be continual improvement in control, because it meant that at any time the standard of control at the experimental establishments was the standard of the best, if they assumed, as they must, that the pilots were using their experience and knowledge in selection.

Mr. R. McKinnon Wood (Principal Scientific Officer, R.A.E.): With regard to Mr. Stevens' suggestion that an aeroplane might be regarded as satisfactory in regard to control of spin if it would come out of a spin in three turns, he expressed the view that a higher standard should be aimed at. He urged that view not because a machine which would not come out of a spin in less than three turns was necessarily dangerous, but because spinning was so variable a phenomenon, and it was possible that an aeroplane which, in certain tests at a testing station, came out of a spin in from $2\frac{1}{2}$ to $3\frac{1}{2}$ turns, might not come out at all on a subsequent occasion, even though it were in the hands of quite an experienced pilot. For the character of the spin developed appeared to depend very much in many cases upon the manner in which the spin was entered, and it was very difficult to cover all kinds of spins in the course of tests at a testing station, where there was considerable pressure of work.

Mr. M. L. Bramson (Member of Council): He suggested that an incidence indicator would be of considerable use, for it seemed to him that the angle of incidence at which an aeroplane flew at any particular moment was a true characteristic of the condition of the machine, and did not—as did the air speed, for instance—have a different meaning depending on the conditions of loading in which the machine was flying, whether dynamic or static. It might be slightly irrelevant, although not wholly, to consider the case of an aeroplane gliding in to land. The aim of the pilot was to ensure some reasonable margin above the stall. Normally, he thought of the stall in terms of speed, forgetting that any given aeroplane would stall at very different speeds, depending on both the static and dynamic loading. If, instead of that, he were to glide in with a reasonable margin of incidence above the stalling incidence, or rather, below the stalling angle, then he would be flying consistently. This had a bearing upon what Mr. Stevens had referred to when he had said that a simple test might be the measurement of the smallest circle around which an aeroplane could fly. Would not that depend on the pilot, unless one defined the conditions under which he must fly around that circle? If one used the incidence indicator, and knew the angle of incidence corresponding to the maximum lift coefficient of the machine as a whole, and if the pilot performed the quick turn at that incidence, one would get a true comparison between various aeroplanes.

Dr. A. P. Thurston: He thought that no apology was required if he referred to methods of control which would be instinctive or automatic and in which the response of the aircraft would be proportioned to the control movement.

It was obvious that a control surface, if placed before a main plane, could be made to operate in accelerated air whereas if placed behind the main plane, it was bound to run in decelerated air, hence for this reason alone it would appear desirable to place the control surfaces in front of the wings. In addition control or rider planes in front of the main plane were in a position to set up the maximum interference with the flow over the main plane and thus to exercise a maximum degree of control with the minimum area and frictional loss.

Apt or Alula control took advantage of these facts and used rider planes in front of the main plane to act both as ailerons and rudders. Aileron effect was obtained by varying the gap and rudder effect by altering the inclination. This type of control had the further advantage that it might be used to obtain lateral control when the main plane was stalled, and also to prevent the main planes from stalling.

Another form of control which was automatic in its action and which could be used to prevent stalling and to give more or less " vertical " flight, was the

rotor thumb control. In this control the rider planes were mounted on the top front edge of the main plane, as shown in Fig. 1, so as to be capable of rotation about an axis approximately normal to the chord of the main plane, and to rest closely against the nose of the said plane at normal angles of flight. It was found that a rider plane so mounted set itself at an inclination to the front edge,

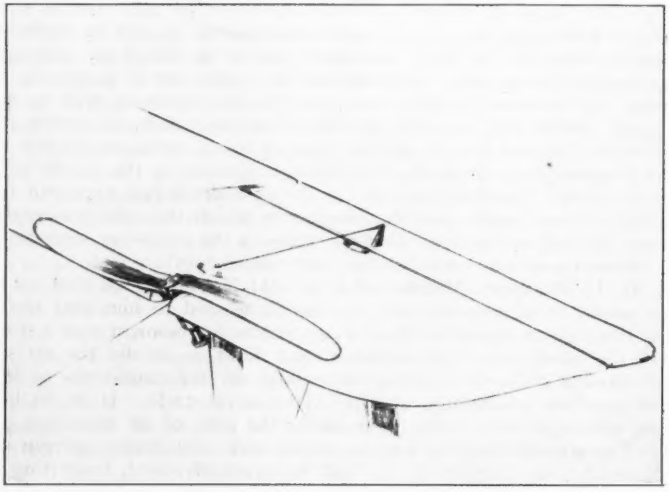


FIG. 1.

when the stalling angle was approached, and gave an " anti-stalling " effect and rotated rapidly, and rose on its spindle, giving a strong lifting effect, at the nose of the main plane, when the inclination of the main plane was still further increased.

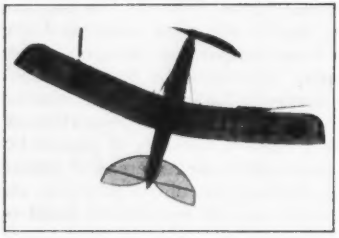


FIG. 2.

Figs. 2 and 3 show a model in flight fitted with such rotors.

Fig. 2 shows the port rotor just coming into action with the starboard rotor already in rotation. Fig. 3 shows the machine flying at an angle much greater than the normal stalling angle with both rotors in rotation.

A strong lateral control was obtained either by controlling the inclination of the axis of rotation or by varying the resistance to rotation of the rotors.

The lift curve of a main plane fitted with these rotors was found to increase up to angles of 34°, if the axis of rotation was suitably inclined. Such a device

gave in effect a combined aeroplane and autogiro with the advantage of a much greater power of control than is possessed by either of these types.

FIG. 3.

Major T. M. BARLOW (Member of Council) (*communicated*): The lecturer has certainly tackled a controversial matter in an able way. He would be pleased to know, however, which opinions expressed are his own and the Air Ministry, especially the paragraph where he refers to the aeroplane getting like a Christmas tree—that seems an advance on official lines in the right direction.

Another point for interest is the " diving " test. As is well-known, many continental built aircraft either cannot be dived at high speeds because of fear of break up of wings or controls or hunt so badly fore and aft as to be useless as a gun platform. It would be useful to have this paragraph elaborated as there is no doubt the British official diving tests are of considerable worth in judging the efficiency of aircraft of the single-seater fighter type.

He hoped that development of force measuring devices on the lines of control wire force indicators and recorders will be pushed. This to his mind is the right way to tackle the subject, and research should be given first priority in control experiments. They started a scheme at Martlesham several years ago, but the importance was not officially realised and the matter dropped as research at that time was not part of the Experimental Station duties.

In his opinion the best of the devices for measuring the flight path speeds, etc., is a combination of theodolite and cinema camera. Such apparatus has recently been perfected by a well-known German instrument company and it provides for accurate times and position of all movements with a photographic

record of the aircraft itself in addition to the watch and theodolite settings. This with the force recording apparatus will help considerably in the solution of this difficult problem of controls. The accuracy of these instruments as regards speed is said to be of the order of 0.5 metre/sec.

The lecturer's fifth lesson from experience he thought requires revision, certainly a better definition as a check on the majority of American aircraft will show the centre of the tail plane almost on the perimeter of the circle if the diameter is taken as the leading edge of the plane (top plane in case of a biplane) whilst if the diameter is taken midway between the spars the tail unit is inside this circle. He was not by any means objecting to Mr. Stevens' statement preferring long fuselages but only to his interpolation of the American rule.

Dr. LACHMANN (*communicated*): He fully agreed with Mr. Stevens that slots are no cure for the flat spin or for recovery from spinning in general. It is, as a matter of fact, unjustifiable to expect it otherwise.

Some time ago a series of experiments were carried out for Messrs. Handley Page, Ltd., by the Gottingen Aerodynamic Institute with a slotted R.A.F. 34 aerofoil. Lift and drag was measured over the complete range of angles, *i.e.*, from 0 to 90°. The object of these tests was, in the first place, to obtain data for the calculation of autorotation of slotted wing sections at large angles of incidence, and this main object was combined with the hope of developing something like a flat top curve for the normal forces (perpendicular to the chord). This hope was not fulfilled. However, very valuable information in regard to the behaviour of slotted wings in flat spins could be obtained from those experiments.

Diagram Fig. 1 represents the K_N curves for several aerofoils, Gottingen

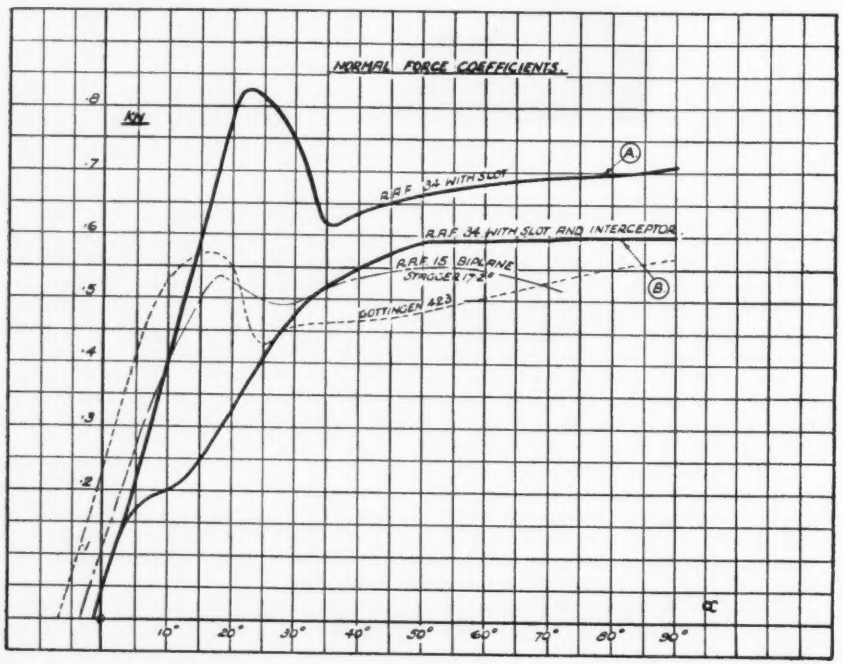


FIG. 1.

423 monoplane,* R.A.F. 15 biplane* and R.A.F. 34 monoplane with complete slots (Curve A).

The K_N curve for Gottingen 423 predicts autorotation at large angles of incidence, in fact autorotation at large angles can be expected quite generally for all high lift sections, owing to the pronounced peak of the K_N curve. This also applies to the slotted R.A.F. 34 and probably to all slotted aerofoils, as their investigations with various slotted arrangements for this section left scarcely any hope of obtaining a flat top K_N curve.

The diagram in Fig 1 also contains a K_N curve for a slotted aerofoil plus interceptors (Curve B). It becomes quite evident that a very strong damping in roll at large angles of incidence must be obtainable by raising the interceptor on the upgoing wing, as the interceptor cuts off the peak of the K_N curve. Supposing the port wing drops and the starboard wing rises, and the interceptor is brought into position on the starboard wing, the angle of incidence being say 50°, according to the diagram one will then obtain an increase of angle of attack and of K_N correspondingly on the port wing (A) and a reduction of angle and K_N at the starboard side (B), in other words, a damping couple is produced. The magnitude of this couple is considerably large owing to extensive calculations based upon the data as represented in Fig. 2.

The theoretical conceptions were confirmed by wind tunnel experiments in the H.P. 4 wind tunnel. A model of the slotted biplane was so arranged that it could freely rotate around an axis passing through its C.G., this axis being parallel to the flow. When arranged at an angle of incidence of 50° or more the wing began to autorotate with very high revolutions. They then arranged a little interceptor plate on the upcoming wing and artificially produced the same rotation as before. After releasing the wing rotation damped out completely and reversed its direction.

Mr. Stevens is of opinion that the addition of an interceptor makes a Christmas tree out of the aeroplane. He believed that if complete damping of autorotation at large angles of incidence can be realised by means of interceptors, one is well justified to say that the interceptor would then act rather as a Santa Claus to a certain class of aeroplane. Besides, they had been able to develop a very simple interceptor device (Fig. 2). The interceptor plate (*a*) is operated by a bell crank lever (*b*) which is pivoted to the rear link and controlled by the aileron through the cable (*c*). If the slot is closed the interceptor is out of action and the cable (*c*) is loose. In the open position of the slot the cable is tautened and the interceptor is raised. The gear ratio between the interceptor movement and aileron movement is fairly high, so that an angular movement of 90° for the interceptor corresponds with an aileron movement of about 60°. Further movement of the aileron after the interceptor has been fully raised is taken up by a spring (*d*) or some other lost motion device.

He believed that it was not sufficient for the sake of safety to concentrate upon the lateral stability only, neglecting improvement of longitudinal control in a stall. Although wing tip slots, when correctly arranged and properly shaped, give perfect lateral stability and provide a flat top lift curve for the total aeroplane, they cannot prevent that nasty dip of the nose and the short dive which every aeroplane takes in a more or less pronounced way when stalled before it settles down to a steady stalled glide.

He thought it necessary to pay more attention to the improvement and the proper placing of the tail control, as some recent investigations which they had carried out indicated that on some aeroplanes a very strong wake is produced behind the wings and within this wake the velocity is reduced by about 20 to 30 per cent. corresponding to the loss of dynamic pressure and tail efficiency of 40 to 60 per cent.

* From Z.F.M., 1930, No. 1.

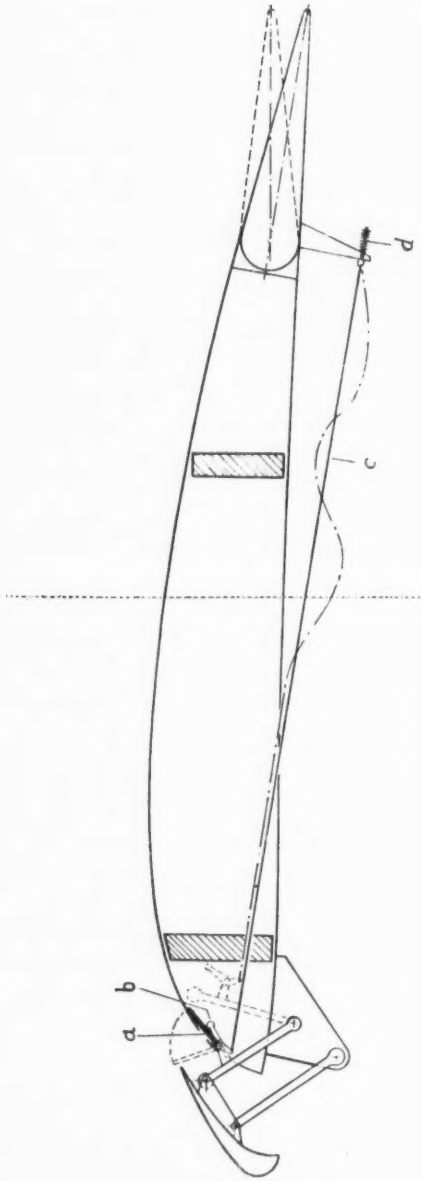


FIG. 2.

Low wing monoplanes with short bodies and aeroplanes fitted with flaps appear to be worst.

Reply to Discussion

Replying to the President's remark that apparently he had considered only two ways of assessing the value of controls, namely, by the use of elaborate instruments, on the one hand, and by recording pilots' opinions, on the other hand, he had hoped to have made it clear that, although that was more or less the case at the moment, he looked forward to introducing at Martlesham some test such as the performing of the figure of 8, or some means of measuring turn and expressing the results in figures. He wished particularly to emphasise that we must ensure that the things we measured did mean something, and a good deal of thought would have to be expended in that direction before it was decided what to measure. We had not yet the simple instruments we needed, but he believed they would be available in a few months; then various forms of manœuvre could be tried and we should be able to ascertain whether or not the figures obtained for the various machines would place those machines in the order of merit in which the pilots themselves placed them. As to the President's suggestion that pilots were apt to become accustomed to particular types of control and to regard anything new with disfavour, at Martlesham the changes were so frequent that the pilots were not likely to get into a groove.

With regard to Squadron Leader England's statement that, although an aeroplane might not drop a wing at the stall, it might be vicious at the stall, his instruments were used in an endeavour to find out whether that happened or not. He did not necessarily agree that it was a bad thing to be unable to stall an aeroplane. After all, all that was lost by not actually stalling was the increase in drag at from 12 to 14 degrees incidence, and that was a small quantity compared with what could be done with brakes. As to the interceptor, he had merely stated that it was complicated; it had done all that he had required of it. He was glad to know, however, that the complications had been overcome.

Replying to Squadron Leader Bennett-Baggs, he did not know how they were to decide which were the right forces to measure, and what relative value the forces should have in order to produce harmony. Even though various things were measured, it was difficult to decide which were the right things to measure, and under those circumstances all that could be done was to measure certain things and to compare their judgments with pilots' opinions as to which aircraft were the best. As to the question of whether or not the rudder bar worked in the proper direction, he was afraid that Squadron Leader Bennett-Baggs did not bank correctly if he found that all his weight went on to either one foot or the other. Only a few days ago he had been arguing this matter with a well-known pilot, who, in demonstrating the movements of the body when one turned to the left, had stuck out his right foot. That indicated how instinctive it was. It must not be thought that he wished to make a change in regard to rudder control at this stage; he had merely stated that at present it did not fit in with what he would like to do logically, and he believed that it caused difficulty to those who were learning to fly—at any rate, he had had difficulty in this respect. To illustrate the point further, he referred to the box carts used by small boys, and which were steered by placing the feet on the front axle; he had never seen a boy steering with his legs crossed!

Referring to the formula for testing controls, mentioned by Squadron Leader Haig, he thought that the mathematicians had proved it to be wrong, so that its use had been discontinued. As to the point that the use of too many gadgets for the purpose of improving control might result in affecting the performance of a machine adversely, it was a question of weighing up relative values and preserving a balance.

He agreed with Mr. Irving's remarks concerning the blanking of fins and rudders by tail planes. As to his statement that research on controls was largely developed to the end of finding out how nearly model and full-scale aeroplanes agreed, his point was that, once they had decided what they wanted an aeroplane to do, the wind tunnel experimenters and mathematicians could say whether or not a particular arrangement of controls would do it—provided the full-scale machine did what the model did. The first essential was to decide whether or not that applied to controls as to other things.

Dealing with Flight Lieutenant Cahill's remarks concerning the rudder controls, if one were piloting an aeroplane on a vertical bank it was surely more normal to push on the top half of the rudder bar to get the nose down than to push on the lower half. He agreed with Major Buchanan as to the necessity for ensuring that controls were not dangerous. That was a matter of finding out from pilots which forces were safe and which were dangerous.

As to the suggestion by Mr. McKinnon Wood that an aeroplane should come out of a spin in less than three turns, he said that the adoption of a standard of less than three might result in condemning too many aeroplanes.

Replying to Mr. Bramson's question as to whether the incidence indicator would be a useful instrument, he did not think it had much to do with the purpose under discussion, although it might be useful in certain circumstances.

In reply to Dr. Thurston, as regards the rotary thumb control, perhaps Dr. Thurston had produced the new form of control he had asked for without all the complications. If so, he wished him luck.

In reply to Major Barlow, all the opinions expressed are my own. I agree that the diving test is very valuable and it would be useful to have it adopted internationally. The simple force measuring devices will be developed as rapidly as possible. As regards apparatus for measuring flight path, the theodolite and cinema camera are probably quite good, but a cinema camera obscura might be better still. As regards the interpretation of the American rule for length of fuselage, I have always understood that the circle should be struck with its centre at the C.G. of the aircraft.

In reply to Dr. Lachmann, I am glad he agrees that slots alone are no cure for the flat spin. The slot and interceptor would appear from his diagrams to be very good and the latest form of this device shown in his Fig. 2 certainly appears simple enough.

I also agree that longitudinal control at the stall will repay investigation.

THE EQUILIBRIUM OF LOADED IMPELLER BLADES WITH SPECIAL REFERENCE TO THE STRENGTH OF THIN METAL AIRSCREWS

BY

THE LATE PANDIA ANTONIO RALLI

(Some time Head of the Technical Staff of the Fairey Aviation Co. Ltd.)

(Theory developed during the years 1928-1929.)

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Part I.

Definitions and Notations

1. Co-ordinate Axes

The co-ordinate system used is shown in Fig. 1.

The axis $\overline{OZ}$ coincides with the axis of rotation and the $\overline{XY}$ plane is chosen parallel to the plane of rotation.

The $\overline{ZX}$ plane serves as a longitudinal datum plane for the airscrew blade, and is perpendicular to the planes of the cross-sections of the blade. The axis OX is drawn in the direction of the radius from root to tip.

2. Neutral Surface

The airscrew blade is considered reduced to a " ruled surface," the generators of which are the neutral axes (approximately the minor axes of inertia) of the consecutive blade sections.

These generators are all contained in planes parallel to each other and to the YZ plane. They form with the XY plane an angle ϕ to be referred to as the " pitch angle," which varies continuously along the blade, in accordance with some function of x which, in general, is arbitrary. In addition all the generators rest on a continuous curve contained in the ZX plane whose ordinates are defined by another arbitrary function,

$$H = f(x).$$

The equation of the neutral surface thus defined, expressed in the standard form $z = F(x, y)$, is then

$$z = H + y \tan \phi \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where both H and ϕ are arbitrary functions of x.

The inclination of the generators on the $\overline{XY}$ plane can also be expressed by means of the function

$$p = x \tan \phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where p may be termed the " pitch " of the neutral surface at x. Equation 1 in this case becomes

$$z = H + p\,(y/x) \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The function p has the same degree of generality as ϕ.

3. Centroid Co-ordinates

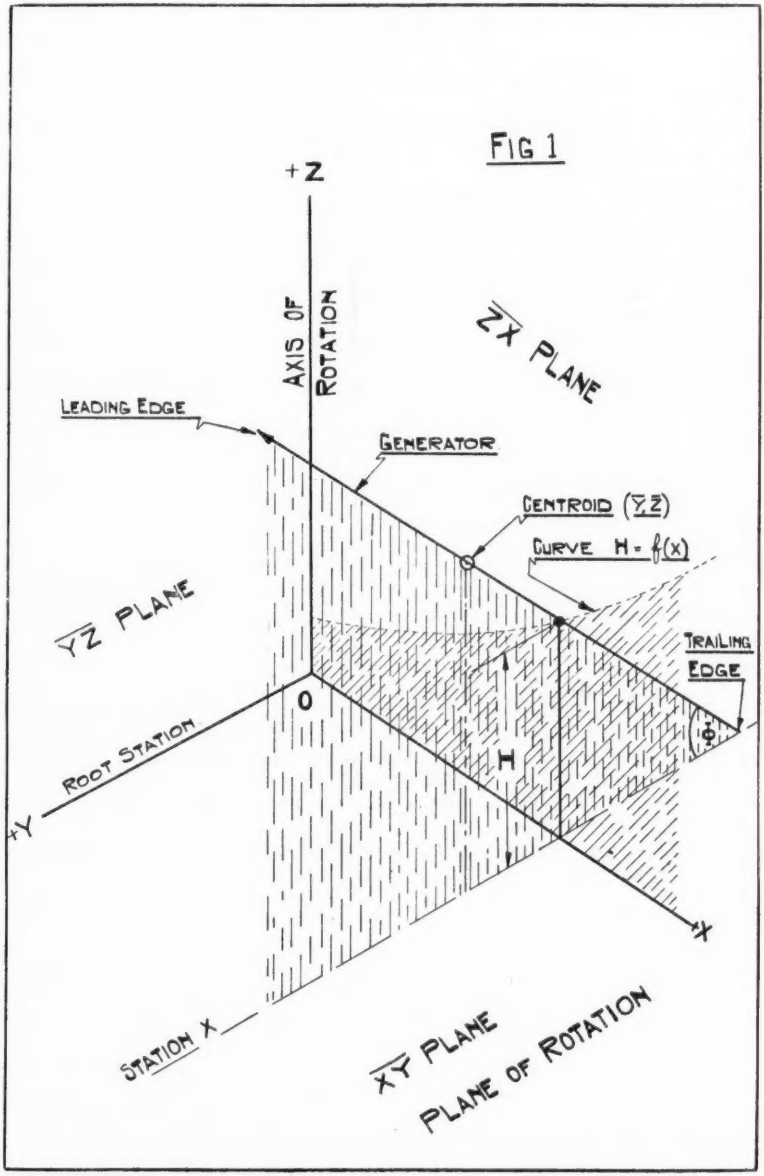
On any generator at a station x there is a point denoted by the co-ordinates $\bar{y}$, $\bar{z}$, which corresponds to the centroid of the section. The centroid being on the neutral axis, its co-ordinates necessarily satisfy the equation of the neutral surface, so that

$$\bar{z} = H + p\,(\bar{y}/x) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is now apparent that all the elements of a blade section are completely located at any station x if the arbitrary functions H and p and one of the centroid co-ordinates $\bar{y}$ or $\bar{z}$ are known.

4. State of Ease

In the unstrained state all the elements of the airscrew blade are given by its working drawing.



Co-ordinate System.

The arbitrary functions and the centroid co-ordinates in this condition will be distinguished by the suffix " o." Equations (3) and (4) then become

$$z_o = H_o + p_o\,(y_o/x) \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

and

$$\bar{z}_o = H_o + p_o\,(\bar{y}_o/x) \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

respectively.

It should be noted that the variables H_o, p_o, $\bar{y}_o$, $\bar{z}_o$, and all their derivatives are known functions of x in a state of ease.

5. *Deformations*

In general, the effect of deformations will be to convert equations (3*a*) and (4*a*) into equations 3 and 4, and the state of equilibrium under load will be determined if the functions H, p, $\bar{y}$ and $\bar{z}$ can be expressed in terms of their initial values H_o, p_o, $\bar{y}_o$, $\bar{z}_o$, and of the resultants of the external forces and moments.

Various restrictions on the generality of the strains will be made, in accordance with the usual practice in the treatment of airscrew blades by the elementary beam theory, namely :—

(*a*) Uniform longitudinal extensions and all strains due to shear will be ignored.

(*b*) Strains due to couples about the axis of X, tending to twist the blade, will not be taken into account.

It follows from the exclusion of torsion that the function p remains unaltered before and after strain, for moderate displacements such as occur here, and the suffix " o " will be abandoned in connection with this variable.

Since rigid body displacements need not be taken into consideration, it is always permissible to assume that the curve " H " passes through the origin of co-ordinates and retains its original slope dH_o/dx, before and after strain, at the root section $x=0$. It may also be noted that the curve H, being by definition the trace of the neutral surface on the $\bar{Z}X$ plane, remains in that plane after deformation, although it will not then consist of the same individual points which occupied that position in a state of ease, nor retain its original shape.

In contrast with the points of the curve H, the points which constitute the locus of centroids will retain their individuality after deformation and their locus will be transformed into a new curve regarding which all that can be stated at this stage is that it will be continuous and constrained to lie on the neutral surface, and that its co-ordinates and component initial slopes at $x=0$ will remain unaltered by the deformations.

The observations regarding initial slopes are equivalent to the statement that the blade is a cantilever built in at its root.

6. *Component Strains at a Given Station*

In accordance with the definitions and assumptions of the previous paragraphs, the only external couple capable of producing strain, at any radius x, is the sum of the moments about the neutral axis at x of all the external forces acting on one side of that section; this may be termed the " effective " bending moment at the station (*i.e.*, effective in producing strain).

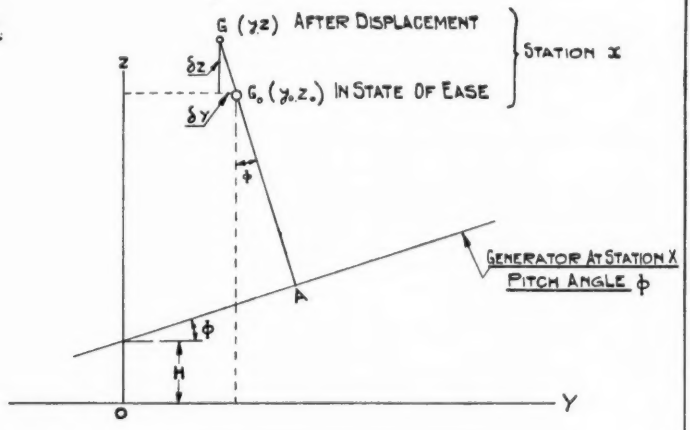
Under the influence of this moment (torsion being neglected) the blade will " fold " through a small angle $\delta\theta$ about the generator acting as a hinge, and all blade elements outside the station considered will experience displacements consistent with this rotation.

Imagine then, any arbitrary curve drawn on the neutral surface from root to tip, and consider a station x at which the co-ordinates of a point on this curve in a state of ease are y_o and z_o, the pitch angle at x being ϕ; and let $\delta\theta$ be a small rotation occurring at a station nearer the root than x, say X, at which the pitch angle is Φ.

Then due to the rotation $\delta\theta$ at X, the point y_o, z_o, at x will experience a small displacement, such that (Fig. 2)

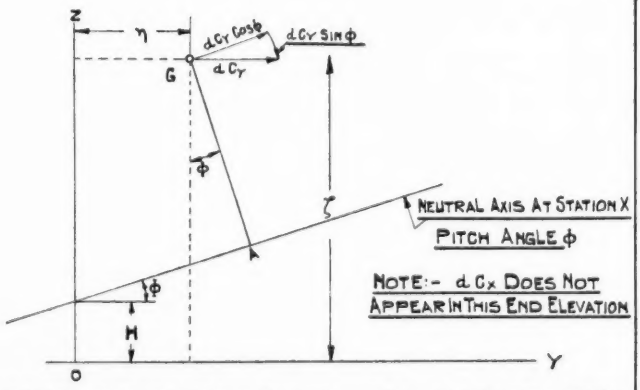
$$\delta z = (x - X) \cos \Phi \delta\theta$$
$$\delta y = -(x - X) \sin \Phi \delta\theta.$$

FIG 2.



COMPONENT DISPLACEMENTS OF A POINT G_o AT STATION X DUE TO A BENDING STRAIN ON A GENERATOR AT AN INNER STATION X.

FIG 3.



NOTE:-

FIGS 2 AND 3 ARE DRAWN FOR A LEFT HAND TRACTOR LOOKING OUTWARDS FROM BOSS TO TIP

MOMENTS ABOUT A NEUTRAL AXIS AT STATION X OF A CENTRIFUGAL FORCE ELEMENT dC ACTING AT THE CENTROID OF A SECTION SITUATED AT A STATION § OUTSIDE X

System of Notation.

If all intervening generators between the stations $X=o$ and $X=x$ contribute small rotations at the rate $\delta\theta/dX$, $\delta\theta$ being in general different at each generator, the total displacement of the point y_o, z_o, at x will be

$$z - z_o = \int_{X=o}^{X=x} (x - X) \cos \Phi \, (\delta\theta/dX) \, dX$$

$$y - y_o = -\int_{X=o}^{X=x} (x - X) \sin \Phi \, (\delta\theta/dX) \, dX.$$

Differentiating with respect to x, and noting that there will be two terms in the second member since x appears under the integral, we obtain :—

$$d\,(z - z_o)/dx = [(x - X) \cos \Phi \, (\delta\theta/dX)]_{X=x} + \int_{X=o}^{X=x} \cos \Phi \, (\delta\theta/dX) \, dX$$

and a similar relation for $d\,(y - y_o)/dx$; but since for $x=X$, $x - X = o$, these reduce to :—

$$\begin{cases} dz/dx = dz_o/dx + \int_o^x \cos \Phi \, (\delta\theta/dX) \, dX \\ \\ dy/dx = dy_o/dx - \int_o^x \sin \Phi \, (\delta\theta/dX) \, dX. \end{cases}$$

Differentiating again, and noting that since in this case there are no functions of x under the integral, differentiation is performed by removing the integral sign and making $X=x$ in the integrand, we obtain :—

$$\begin{cases} d^2z/dx^2 = d^2z_o/dx^2 + \cos \phi \, (\delta\theta/dx) \\ d^2y/dx^2 = d^2y_o/dx^2 - \sin \phi \, (\delta\theta/dx) \end{cases} \quad . \quad . \quad . \quad (5)$$

where ϕ is the value of Φ at x.

7. *Moment of Internal Forces*

In accordance with elementary theory the moment of the internal forces is given by the expression $EI\,(d\theta/dx)$, where EI is the flexural rigidity and the element dx is a permissible approximation to the projection of the line element ds for beams of easy curvature.

Denoting by M the effective bending moment at x due to all the external forces to the right of x, we have for equilibrium of moments :—

$$EI\,(d\theta/dx) = M$$

and, substituting in equation (5) :—

$$\begin{cases} d^2z/dx^2 = d^2z_c/dx^2 + (M/EI) \cos \phi \\ d^2y/dx^2 = d^2y_o/dx^2 - (M/EI) \sin \phi \end{cases} \quad . \quad . \quad . \quad . \quad (6)$$

which establishes a relation between component strains and effective bending moment at any station x.

It should be noted that since M/EI and ϕ are constant all along a generator, equation (6) is valid for the component line elements at x of any arbitrary curve drawn on the neutral surface in a state of ease, consequently also for the locus of centroids.

This equation indicates how the initial component curvatures in the planes $\overline{ZX}$ and $\overline{XY}$ (elevation and plan) of an arbitrary line drawn on the surface in a state of ease are modified by the components of the effective bending moment at the station.

If M/EI were a known function of x only (air loads), (6) would be a simple differential equation for z and y when z_0 and y_0 are given; actually, on account of the centrifugal forces M depends on the displacements of z and y at all points of the blade outside x, and further developments are necessary before equation (6) can be utilised.

8. *Moment Due to Centrifugal Force*

In order to express clearly the effect of the centrifugal force it will be necessary to consider simultaneously two stations on the blade at a finite distance from each other, and to distinguish certain identical variables at the respective stations by different notations.

For this purpose, let A (Fig. 3) be the station about whose neutral axis moments are being calculated and K, at a greater radius than A, a station at which an element of centrifugal force producing the moments, is situated; respective variables will be distinguished by the symbols in the following table:—

	Inner Station A.	Outer Station K.
Abscissa of station	x	ξ
" Z " co-ordinate of centroids	z^*	ζ
" Y " " "	y	η
Pitch function	p	w
Pitch angle	ϕ	ψ
" H " curve in plane ZX	H	h

The element of centrifugal force dC at K is directed along a line joining the centroid at K to the axis OZ and running parallel to the XY plane. The components are:—

$$\begin{cases} dC_x = dC/\sqrt{[1+(\eta/\xi)^2]} = dC \text{ (approx.)} \\ dC_y = dC\,(\eta/\xi)/\sqrt{[1+(\eta/\xi)^2]} = dC\,(\eta/\xi) \text{ (approx.)} \\ dC_z = 0. \end{cases}$$

The arm of the moment of dC is the perpendicular distance between the line of action of dC and the neutral axis at A. Taking the components separately, the arm of dC_x is:—

$$a_x = (\zeta - H)\cos\phi - \eta\sin\phi$$

The component dC_y must be further resolved into a component " $dC_y \cos\phi$ " which is parallel to the neutral axis at A and therefore has no moment, and " $dC_y \sin\phi$ " whose arm is:—

$$a_y = \xi - x$$

The element of moment at a station x, due to a single element of centrifugal force at a station ξ, is therefore:—

$$dm = dC\,[(\zeta - H)\cos\phi - \eta\sin\phi + (\eta/\xi)(\xi - x)\sin\phi],$$

which reduces to:—

$$dm = dC\,[\zeta\cos\phi - x\sin\phi\,(\eta/\xi) - H\cos\phi].$$

If the element dC varies from the station $\xi = x$ to $\xi = l$ (tip station) at the rate $dC/d\xi$, the total moment at x due to all the centrifugal force elements acting between x and the tip is:—

$$m = -\int_l^x (dC/d\xi)\,[\zeta\cos\phi - x\sin\phi\,(\eta/\xi) - H\cos\phi]\,d\xi \quad . \quad . \quad (7)$$

where l is the value of x at the tip (tip radius). We observe that this moment

* As no other co-ordinates but those of the centroids are used throughout the remainder of this work the " barred " symbols will be abandoned.

is a function of the position of all the centroids between x and the tip, but a function of ϕ and H at x only. It should be particularly noted that H and ϕ are not functions of ξ in this expression.

9. *Air-load Moments*

The air load moments are obtained by standard methods which need not be described. Denoting by M_T and M_Q the moments due to thrust and torque respectively, the effective moment resolved along the neutral axis is:—

$$X = M_T \cos \phi + M_Q \sin \phi \quad . \quad . \quad . \quad . \quad (8)$$

Only the effective moment X is required for the present purpose; it remains a known function of x which is not varied by deformations.

Part II.

Equilibrium of Loaded Blade for Airscrews of Arbitrary Pitch Distribution (General Case)

10. *Introduction*

Equations of equilibrium under load and a differential equation for the deformed blade can now be obtained by equating the internal and external moments at every station.

In the present section the analysis will be kept free from any particular assumption of pitch distribution, up to the statement of the differential equation. A brief discussion of this equation is given and the effect of introducing constant pitch indicated. As this subject is treated independently in Part III, it is unnecessary to read Part II for the purpose of applications.

In the most general case, where H and p are any functions whatsoever of x in a state of ease, there is apparently no practicable solution of the differential equation not involving trial and error. It has been observed, however, that for the great majority of airscrews the function "p" is practically a constant over at least the outer two-thirds, and, for many other types, the whole length of the blade.

This observation will be taken advantage of in Part III of this investigation, over the area where it applies with sufficient approximation, and it will be seen that a rigid direct method, free from trial and error, can then be developed. For those parts of any airscrew where the approximation does not hold, almost invariably at the root stations, a simple extension of the method enables the stresses to be determined with an error on the safe side, which should be sufficient for all practical purposes.

It is also obvious that a rigid check is always available by comparing the moments due to curvature with the moments of the external forces after any solution has been obtained numerically.

A numerical example is given applying to a Fairey "D" type airscrew, on which the operation of the method on the constant pitch portion of the blade and on the variable pitch root can be studied.

11. *Equations of Equilibrium*

For equilibrium of internal and external moments at any station x we have the simultaneous equations (6) (para. 7) and the relation:—

$$M = -\int_{\lambda}^{x} (dC/d\xi) \left[\zeta \cos \phi - x \sin \phi \, (\eta/\xi) - H \cos \phi\right] d\xi - X \quad . \quad . \quad (9)$$

which expresses the sum of the moments of the external forces at a station x, the symbol X denoting the effective air load moment defined in paragraph 9.

Dividing throughout by $\cos\phi$, which is a function of x only and not of ξ, and introducing the function $p=x\tan\phi$, we obtain :—

$$M/\cos\phi=-\int_{l}^{x}(dC/d\xi)\,[\zeta-p\,(\eta/\xi)-H]\,d\xi-X/\cos\phi.$$

Integrating by parts with respect to ξ, between the limits $\xi=l$ and $\xi=x$ leads to :—

$$M/\cos\phi=-[C\,\{\,\zeta-p\,(\eta/\xi)-H\,\}]_{l}^{x}+\int_{l}^{x}C\,[d\zeta/d\xi-p\,\{\,d\,(\eta/\xi)/d\xi\,\}]\,d\xi-X/\cos\phi.$$

But for $\xi=l$ the centrifugal force (at the tip) is zero, and for $\xi=x$ we have $\zeta=z$ and $\eta/\xi=y/x$ and the expression :—

$$z-p\,(y/x)-H$$

vanishes for $\xi=x$ by virtue of the equation of the neutral surface, (3) (para. 2).

We have in consequence :—

$$M/\cos\phi=\int_{l}^{x}C\,[d\zeta/d\xi-p\,\{\,d\,(\eta/\xi)/d\xi\,\}]\,d\xi-X\cos\phi\,. \qquad (10)$$

Applying equation (6) (para. 7) to the locus of centroids at any station X, we have :—

$$\begin{cases} d^2\,(Z-Z_o)/dX^2=(M/EI)\cos\Phi \\ d^2\,(Y-Y_o)/dX^2=-(M/EI)\sin\Phi \end{cases}$$

and the relation :—

$$d^2\,(Y-Y_o)/dX^2=-\tan\Phi\,\{\,d^2\,(Z-Z_o)/dX^2\,\}\,.$$

Multiplying each side by X and integrating the left hand side by parts, between $X=o$ and $X=\xi$, gives, since $Y-Y_o$ and $d\,(Y-Y_o)/dX$ vanish for $X=o$:—

$$\xi\,\{\,d\,(\eta-\eta_o)/d\xi\,\}-(\eta-\eta_o)=-\int_{o}^{\xi}X\tan\Phi\,\{\,d^2\,(Z-Z_o)/dX^2\,\}\,dX$$

where the series of symbols X, Z and Φ are retained under the integral sign, as on previous occasions, to avoid confusion between the variables to be integrated and those which can be directly expressed in terms of the upper limit of integration.

Dividing throughout by ξ^2, we obtain :—

$$(1/\xi)\,\{\,d\,(\eta-\eta_o)/d\xi\,\}-(\eta-\eta_o)/\xi^2=-(1/\xi^2)\int_{o}^{\xi}X\tan\Phi\,\{\,d^2\,(Z-Z_o)/dX^2\,\}\,dX$$

which is equivalent to :—

$$d\,[(\eta-\eta_o)/\xi]/d\xi=-(1/\xi^2)\int_{o}^{\xi}X\tan\Phi\,\{\,d^2\,(Z-Z_o)/dX^2\,\}\,dX$$

or :—

$$d\,(\eta/\xi)/d\xi=-(1/\xi^2)\int_{o}^{\xi}P\,\{\,d^2\,(Z-Z_o)/dX^2\,\}\,dX+d\,(\eta_o/\xi)/d\xi \qquad (11)$$

where P is written for $X\tan\Phi$, and represents the same arbitrary function of X as p is of x.

Equation (11) can now be used to eliminate the term $d(\eta/\xi)/d\xi$ from equation (10), and the first of equations (6) to eliminate M by writing :—

$$M/\cos\phi=(EI/\cos^2\phi)\ \{d^2(z-z_o)/dx^2\}.$$

Introducing the difference of component slope in the $\overline{ZX}$ plane before and after strain, of the locus of centroids, by the definitions :—

$$\left.\begin{array}{l} d(\zeta-\zeta_o)/d\xi=\sigma \\ d(Z-Z_o)/dX=S \\ d(z-z_o)/dx=s \end{array}\right\}$$

according to the order of integration of the variables considered, we obtain the following equation for this new variable by substitution in equation (10) :—

$$(EI/\cos^2\phi)(ds/dx)=-\int_i^x C\left[\sigma+(p/\xi^2)\int_o^\xi P(dS/dX)\,dX\right]d\xi+X/\cos\phi$$

$$-\int_i^x C\left[d\zeta_o/d\xi-p\{d(\eta_o/\xi)/d\xi\}\right]d\xi \qquad . \quad . \quad . \qquad (12)$$

A solution for s of this equation, which presents itself at this stage in "integral" form for the variable ds/dx in terms of x, would give the complete solution for the shape of the deformed neutral surface whose original shape was determined by the equation :—

$$z_o=H_o+p(y_o/x)$$

and also for the effective bending moment at any station, which would be given by :—

$$M/\cos\phi=(EI/\cos^2\phi)(ds/dx).$$

There is no restriction in equation (12) on the generality of the functions p, z_o and y_o, and the two last terms on the right, representing respectively the air load moments and the centrifugal force moments due to the initial component slopes of the blade in a state of ease, are known functions of x.

Grouping the latter in a single symbol K and writing μ for ds/dx or $(M/EI)\cos\phi$, and θ for $EI/\cos^2\phi$ to simplify notations, the general equation (12) appears in the form :—

$$\theta\mu=-\int_i^x C\int_o^\xi \mu dX\,.\,d\xi-p\int_i^x C/\xi^2\int_o^\xi P\mu dX\,.\,d\xi+K \qquad . \quad . \qquad (12a)$$

It is possible by successive differentiation and appropriate grouping, to reduce this expression to a linear differential equation of the fourth order in μ, the coefficient of each derivative containing from three to four terms, each consisting of known functions of x. But such a form is beyond the scope of practical applications.

By trial and error, the procedure would consist in calculating and tabulating first the known function K at each station. A curve of μ against x would then be tried and checked by performing the double integrations indicated by the two double integrals of the second member; a successful trial would result in these being found reasonably equal to the terms $\theta\mu-K$.

An airscrew so exceptional in pitch distribution as to require treatment by this method would be of such rare occurrence that the subject will not be developed further, and the effect of constant pitch will now be briefly examined prior to the independent investigation in Part III.

12. *Effect of Constant Pitch*

The pitch function having been defined as :—

$$p = x \tan \phi$$

we observe that the function V/nD follows the law $x \tan \phi_0 = \text{constant}$ exactly, ϕ_0 being the angle $\tan^{-1}(V/\pi nd)(D/2x)$.

The angle ϕ will generally be larger than ϕ_0 and will differ on account of relatively small allowances made for incidence and inflow over the working portion of the blade.

Near the root only ϕ may fall below ϕ_0 either to allow for boss interference or because structural difficulties impose a limit to further increase in ϕ.

It will therefore be taken into account that constant pitch can only be assumed from a certain station " τ " outward, but that τ does not generally exceed one-third of the radius. The actual value of τ is settled by inspection of each particular case.

We now proceed to show briefly how the introduction of constant pitch affects equation (12*a*). A self-contained analysis based on constant pitch is given in Part III.

If p is a constant we immediately obtain by differentiating equation (12*a*) :—

$$d(\theta\mu)/dx = C\int_o^x \mu dX + p(C/x^2)\int_o^x P\mu dX - dK/dx$$

$$d(\theta\mu)/dx = C\int_o^x \mu dX + C(p^2/x^2)\int_o^x \mu dX - dK/dx$$

$$d(\theta\mu)/dx = C(1 + p^2/x^2)\int_o^x \mu dX - dK/dx$$

or, since $1 + p^2/x^2 = 1/\cos^2\phi$ and $\int_o^x \mu dx = s$

$$\theta(d^2s/dx^2) + (d\theta/dx)(ds/dx) - (C/\cos^2\phi)\, s = -dK/dx$$

We have now obtained a linear differential equation of the second order with variable coefficients which lends itself to a simple solution for practical applications, now to be described and established independently.

PART III.

EQUILIBRIUM OF LOADED BLADE FOR AIRSCREWS OF APPROXIMATELY CONSTANT PITCH

13. *Introduction*

The case for the airscrew of constant pitch distribution will now be established independently of the general analysis in Part II on account of its importance for applications.

A constant pitch distribution implies the relation :—

$$p = x \tan \phi = \text{constant}.$$

The abscissa of the station at which the pitch ceases to be even approximately constant will be denoted by r, and may vary in practice between $x=0$ and $x=1/3$ of the radius.

Apart from the fact that limits of integration may be affected, all the definitions and notations of Part I regarding deformations and expressions for internal and external couples remain valid, whatever the form of the pitch function.

The method of procedure will consist in the first place in assuming that the portion of the blade between the root and r, where the pitch is variable, has been " solidified."

In consequence of this assumption there will be strains at r due to the loads on the constant pitch portion beyond, but no displacements, and the centroid coordinates and component slopes of the locus of centroids at r will remain unchanged during this phase of the deformation.

The solution for the outer portion having been obtained various degrees of approximation are available to treat the root region.

The simplest method, which errs on the safe side, is to allow the root to remain solidified and simply continue the calculation of the external bending moments from r to 0 as for an undeflected blade.

The root sections will then have benefited by that amount of relief which will result from the deflections taken into account between r and the tip. It is clear that such an approximation is safe, for if the root is released and the air loads, for instance, have overcome the centrifugal forces in the outer region, the added slope due to releasing the root will still further reinforce the relief due to the outboard centrifugal forces, whilst the air loads will remain unchanged. If, on the other hand, the centrifugal forces were overbalancing the air loads when the root was solid, due to forward tilt in a state of ease, the added slopes due to releasing the root would be in the direction of reducing the centrifugal force arms and favouring the air loads.

The same reasoning is in fact followed as in the method of stressing in which deflections are completely ignored and the stresses are required to be satisfactory without the introduction of moments due to deflection; but whereas in such methods the error, though safe, is very considerable, in the conditions now proposed it becomes confined to a small and comparatively stiff portion of the blade.

For a clear approximation it will be found that it is possible to obtain a solution for the constant pitch portion which is linear in terms of the component slopes at r. The external moments extended from r to 0 will then, if expressed in that form, also appear linear in terms of these slopes and a closer approximation then presents itself by solving for the unknown slopes at r by a process analogous to the " three-moment " method for beams. This procedure can be used without much additional labour and it is obvious that it also gives a closer result, not only for the root, but for the whole blade, since the moments are throughout dependent on the slopes at r. The sequence of slopes between 0 and r, however, will not be quite correct.

Finally, it would be possible to find the unknown slopes by applying the general equation (12a); such a process would be very laborious, and it is submitted, unnecessary. For if any airscrew did not show satisfactory stresses on the first or second of the three methods just described, that is when relief has been obtained by the deflection of at least two-thirds of the outer portion of the blade and part of the root, it would be advisable to strengthen the weaker sections or to readjust the initial tilt accordingly.

We now proceed to develop the method, ignoring Part II, and retaining only the definitions of Part I.

14. Moment of External Forces

Denoting by M, as before, the effective bending moment at any station x, due to all the external forces on the outboard side of x (Fig. 1), we have, for the sum of centrifugal and air moments:—

$$M = -\int_{l}^{x} (dC/d\xi)\,[\zeta \cos\phi - x \sin\phi\,(\eta/\xi) - H\cos\phi]\,d\xi - X$$

where the integral represents the moment of the centrifugal forces and X is the moment of the air loads.

Dividing by $\cos\phi$ which is a function of x but not of ξ, we obtain:—

$$M/\cos\phi = -\int_{l}^{x} (dC/d\xi)\,[\zeta - p\,(\eta/\xi) - H]\,d\xi - X/\cos\phi \quad . \quad . \quad (13)$$

where p is now a constant, and we note that on account of this restriction, the upper limit of integration must not extend beyond r.

Integrating by parts between the limits $\xi = l$ and $\xi = x$

$$M/\cos\phi = -\,|\,C\,\{\,\zeta - p\,(\eta/\xi) - H\,\}\,|_{l}^{x} + \int_{l}^{x} C\,[(d\zeta/d\xi) - p\,\{\,d\,(\eta/\xi)/d\xi\,\}\,]\,d\xi - X/\cos\phi.$$

The first term in the second member vanishes since for $\xi = l$ (tip) $C = 0$, and for $\xi = x$, $\zeta = z$ and $\eta = y$ and the expression

$$z - p\,(y/x) - H$$

is zero, by virtue of the equation of the neutral surface.

The effective bending moment then reduces to:—

$$M/\cos\phi = \int_{l}^{x} C\,[d\zeta/d\xi - p\,\{\,d\,(\eta/\xi)/d\xi\,\}\,]\,d\xi - X/\cos\phi \quad . \quad (14)$$

Now for constant pitch, p is the same at the stations x and ξ and the equation of the surface can be written (see equation (3), para. 2):—

$$\zeta = h + p\,(\eta/\xi)$$

where h is the value of H at ξ.

Hence, by differentiation:—

$$d\zeta/d\xi - p\,\{\,d\,(\eta/\xi)/d\xi\,\} = dh/d\xi$$

and therefore,

$$M/\cos\phi = \int_{l}^{x} C\,(dh/d\xi)\,d\xi - X/\cos\phi \quad . \quad . \quad . \quad (15)$$

15. Moment of Internal Forces

The moments set up by strain at a station X (at a lesser radius than x) are given by equation (6) (para. 7) and can be equated to the effective bending moment in the simultaneous form:—

$$\left.\begin{aligned} d^2\,(Z - Z_o)/dX^2 &= (M/EI)\cos\Phi \\ d^2\,(Y - Y_o)/dX^2 &= -\,(M/EI)\sin\Phi \end{aligned}\right\} \quad . \quad . \quad . \quad (6a)$$

from which we obtain:—

$$d^2\,(Y - Y_o)/dX^2 = -\tan\Phi\,\{\,d^2\,(Z - Z_o)/dX^2\,\}\,.$$

Multiplying both sides by X and integrating by parts between the limits r and ξ, we find :—

$$| X \{ d(Y-Y_o)/dX \} |_{X=r}^{X=\xi} - \int_{X=r}^{X=\xi} \{ d(Y-Y_o)/dX \} dX = -p | d(Z-Z_o)/dX |_{X=r}^{X=\xi}$$

But since the blade is assumed to be " solidified " between the root and $x=r$, it follows that at station r :—

$$(Z)_r-(Z_o)_r=0 \qquad (Y)_r-(Y_o)_r=0$$
$$(dZ/dX)_r-(dZ_o/dX)_r=0 \qquad (dY/dX)_r-(dY_o/dX)_r=0$$

The terms resulting from the lower limit of integration therefore vanish, and there remains :—

$$\xi \{ d(\eta-\eta_o)/d\xi \} -(\eta-\eta_o)=-p \{ d(\zeta-\zeta_o)/d\xi \},$$

or multiplying throughout by p/ξ^2 :—

$$p \{ d[(\eta-\eta_o)/\xi]/d\xi \} = -(p^2/\xi^2) \{ d(\zeta-\zeta_o)/d\xi \} \quad . \quad . \quad (6b)$$

We have also from the first of equations (6a) :—

$$d(\zeta-\zeta_o)/d\xi=\int_r^\xi (M/EI)\cos\Phi dX$$

and from equation (6b) above :—

$$-p \{ d[(\eta-\eta_o)/\xi]/d\xi \} =(p^2/\xi^2)\int_r^\xi (M/EI)\cos\Phi dX.$$

Adding these two last equations we obtain :—

$$d(\zeta-\zeta_o)/d\xi-p \{ d[(\eta-\eta_o)/\xi]/d\xi \} =(1+p^2/\xi^2)\int_r^\xi (M/EI)\cos\Phi dX.$$

By virtue of the equation of the surface with constant pitch, and noting that $1+p^2/\xi^2=1/\cos^2\psi$ we have :—

$$\cos^2\psi \{ d(h-h_o)/d\xi \} =\int_r^\xi (M/EI)\cos\Phi dX \quad . \quad . \quad . \quad (16)$$

16. *Differential Equation of Deformed Blade*

We have now to equate the moments due to internal and to external forces at any station.

It is convenient to introduce an auxiliary variable, analogous to a slope,

$$\left.\begin{aligned} s_o&=\cos^2\psi\,(dh_o/d\xi) \text{ in a state of ease} \\ s&=\cos^2\psi\,(dh/d\xi) \text{ after deformation} \end{aligned}\right\} \quad . \quad . \quad . \quad (17)$$

and the difference :—

$$\sigma=s-s_o=\cos^2\psi \{ d(h-h_o)/d\xi \} \quad . \quad . \quad . \quad (18)$$

and to substitute these in the equations

$$M/\cos\phi=\int_l^x C\,(dh/d\xi)\,d\xi-X/\cos\phi \quad . \quad . \quad . \quad (15)$$

and

$$\cos^2\psi \{ d(h-h_o)/d\xi \} =\int_r^\xi (M/EI)\cos\Phi dX \quad . \quad . \quad . \quad (16)$$

The substitution is :—

$$M/\cos\phi = \int_{i}^{x} (C/\cos^2\psi)\, s d\xi - X/\cos\phi \quad . \quad . \quad . \quad (15a)$$

and

$$\sigma = \int_{r}^{x} (M/EI)\cos\Phi dX \quad . \quad . \quad . \quad . \quad (16a)$$

By differentiation (16*a*) with respect to x, we have :—

$$M/\cos\phi = (EI/\cos^2\phi)\,(d\sigma/dx) \quad . \quad . \quad . \quad . \quad (19)$$

and eliminating $M/\cos\phi$:—

$$(EI/\cos^2\phi)\,(d\sigma/dx) = \int_{i}^{x} (C/\cos^2\psi)\, s d\xi - X/\cos\phi.$$

Differentiating again with respect to x and introducing $s = \sigma + s_0$

$$(EI/\cos^2\phi)\,(d^2\sigma/dx^2) + \{\,\partial\,(EI/\cos^2\phi)/\partial x\,\}\,(d\sigma/dx) - (C/\cos^2\phi)\,\sigma = (C/\cos^2\phi)\, s_0 - \partial\,(X/\cos\phi)/\partial x$$

where the symbol ∂ is used to distinguish derivatives of known functions of x from the derivatives of the unknown function σ.

Dividing throughout by $EI/\cos^2\phi$, we have :—

$$d^2\sigma/dx^2 + (\cos^2\phi/EI)\ \{\,\partial\,(EI/\cos^2\phi)/\partial x\,\}\,(d\sigma/dx) - (C/EI)\,\sigma = (C/EI)\, s_0 - (\cos^2\phi/EI)\ \{\,\partial\,(X/\cos\phi)/\partial x\,\}$$

It will be noted that σ is the unknown dependent variable of this differential equation and that it appears as a function of the independent variable x. All other symbols for the radius, *i.e.*, ξ and X have vanished by virtue of the introduction of appropriate limits of integration.

It is convenient to multiply throughout by l^2 to obtain non-dimensional coefficients, x being now understood to represent x/l, a fraction of the radius. At the tip therefore $x = 1$.

We can also write $\partial\,(X/\cos\phi)/\partial x = S$, analogous to a shear and a known function of x, and by definition, $s_0 = \cos^2\phi\,(\partial H_0/\partial x)$ (equation (17)), which holds for any station x, providing ϕ and H_0 are introduced instead of ψ and h_0.

We thus obtain the differential equation of the deformed blade in the form :—

$$d^2\sigma/dx^2 + (\cos^2\phi/I)\ \{\,\partial\,(I/\cos^2\phi)/\partial x\,\}\,(d\sigma/dx) - (Cl^2/EI)\,\sigma = (Cl^2/EI)\cos^2\phi\,[\partial H_0/\partial x - S/C] \quad . \quad . \quad (20)$$

It has also been found useful for the purpose of applications to define the following non-dimensional coefficients, consisting of constants and known functions of x :—

$$k_0 = C_0 l^2/EI_0 \text{ (analogous to the term } Pl^2/EI \text{ in beams)} \quad . \quad (21)$$

$$a = (C/C_0)\,(k_0/\cos^2\phi) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

$$b = (I_0/I)\cos^2\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

$$Q = \cos^2\phi\,[\partial H_0/\partial x - S/C] \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where C_0 and I_0 are any constants having the dimensions of a force and a moment of inertia respectively.

The differential equation then becomes :—

$$d^2\sigma/dx^2 - (1/b)\,(\partial b/\partial x)\,(d\sigma/dx) - ab\sigma = abQ \quad . \quad . \quad (25)$$

This is a linear equation of the second order in which the coefficients of the derivatives and the second member are known functions of x.

It is dependent for its solution on the discovery of a particular integral when the second member is zero. Subject to this condition a general method is available for establishing the complete integral.

One such particular integral, known (so far as the author is aware) as Lord Kelvin's series, exists in the form of an expansion consisting of a series of multiple integrals the order of which increases by two with every term, but the convergence is slow and the calculations laborious.

An alternative method, now to be described, has been used for the present purpose.

17. The Particular Integral

Denoting by σ_1, the particular solution of (25) when the second member is zero we have the differential equation :—

$$d^2\sigma_1/dx^2 - (1/b)\,(\partial b/\partial x)\,(d\sigma_1/dx) - ab\sigma_1 = 0 \qquad . \quad . \quad (25a)$$

and the expression :—

$$\sigma_1 = \int_1^x b \int_1^x a\sigma_1 dx dx + 1 \qquad . \quad . \quad . \quad . \quad (26)$$

can be described as its equivalent integral form; for differentiating twice we find :—

$$d\sigma_1/dx = b\int a\sigma_1 dx \qquad . \quad . \quad . \quad . \quad (26a)$$

$$d^2\sigma_1/dx^2 = ab\sigma_1 + (\partial b/\partial x)\int_1^x a\sigma_1 dx$$

$$= ab\sigma_1 + (\partial b/\partial x)\,(1/b)\,(d\sigma_1/dx),$$

or, rearranging the terms :—

$$d^2\sigma_1/dx^2 - (1/b)\,(\partial b/\partial x)\,(d\sigma_1/dx) - ab\sigma_1 = 0,$$

which reproduces (25*a*).

Imagine for a moment that σ_1 was known in terms of x, and that equation (26) was to serve as a numerical check on this solution by means of a tabular method of integration.

The blade would be divided into a number of appropriately small intervals " dx " and starting from the lower limit $x=1$ each successive term would be obtainable in terms of the preceding one by performing the operations indicated in equation (26). If the assumed solution for σ_1 had been tabulated in the extreme left hand column of the table it would reappear in the right hand column after the sequence of operations for each station had been performed.

Expressed algebraically, this process carried out between any two neighbouring stations, say $n-1$ and n, when σ_{n-1} is known, is equivalent to solving the following linear algebraic equation for the single unknown σ_n, namely :—

$$\{\,[(a_n\sigma_n + a_{n-1}\sigma_{n-1})\,dx/2 + \int_1^{n-1} a\sigma dx\,]\,b_n + b_{n-1}\int_1^{n-1} a\sigma dx\,\}\,(dx/2) + \sigma_{n-1} = \sigma_n,$$

where a and b are known at all stations and the terms $\int_1^{n-1} a\sigma dx$ and $b_{n-1}\int_1^{n-1} a\sigma dx$ to be denoted by A_{n-1} and B_{n-1} respectively, are known at the station $n-1$ when σ is required to be found at station n.

Solving for σ_n we have :—

$$b_n\,[a_{n-1}\sigma_{n-1}\,(dx/2) + A_{n-1}]\,(dx/2) + B_{n-1}\,(dx/2) + \sigma_{n-1} = \sigma_n\,(1 - a_n b_n dx^2/4) \qquad (27)$$

which enables σ_n to be readily expressed in terms of σ_{n-1}.

All the terms in the left hand member automatically become available as the table proceeds and the term $(1 - abdx^2/4)$ can be tabulated in advance, together with a and b.

The accuracy of the result depends on the size of the intervals chosen. Intervals of $dx = .04$, 3 inches on a 12-foot diameter blade, have been found amply sufficient.

The numerical operations are direct and self-checking, and contain no element of trial and error. The numerical example attached should be referred to for full particulars regarding procedure.

It should be noted that the limits of integration implied in integrating σ_1, from the tip ($x=1$) inwards, are arbitrarily selected for convenience and must not be confused with the two constants of integration of the complete solution. The additive constant 1 in equation (26) has also been chosen for convenience in further developments and applications.

18. *The Complete Integral*

The particular integral σ_1 having been found, we immediately obtain the complete integral in the form:—

$$\sigma = \sigma_1 \int_r^x (b/\sigma_1^2) \int_1^x \sigma_1 a Q dx dx \quad . \quad . \quad . \quad . \quad (28)$$

where a, b, σ_1 and Q are now all known functions of x.

The two constants of integration are correctly determined by the lower limits of integration. For we observe that for $x=r$ we have $\sigma=0$ in accordance with the initial assumption of a solidified root up to $x=r$, where the displacements and changes of slope are zero.

At the tip we find, by examining the first derivative,

$$d\sigma/dx = (b/\sigma_1) \int_1^x \sigma_1 a Q dx + (d\sigma_1/dx) \int_r^x (b/\sigma_1^2) \int_1^x \sigma_1 a Q dx dx,$$

that $d\sigma_1/dx$ is zero for $x=1$ by virtue of equation (26a) of the previous paragraph, and that the first term of the second member is zero as a result of the choice of $x=1$ for the lower limit of the integral. The first derivative of σ is therefore zero for $x=1$, in accordance with the condition that the resultant moment shall be zero at the tip.

It is also possible to obtain a similar solution in terms of the resultant moment. For this purpose σ instead of $M/\cos\phi$ should have been eliminated between equations (15a) and (16a) in paragraph 16.

This procedure has not been followed as it leads to more difficult numerical calculations particularly as regards the variables which have to be tabulated for a state of ease. In the solution for v only the slope dH_o/dx is required, whereas a solution for M involves both dH_o/dx and d^2H_o/dx^2 explicitly.

The plane ZX should be so placed that the curve H_o is as simple as possible. Thus, if the centroids, in a state of ease, are on a straight line perpendicular to OZ, that line can be chosen for the axis of x and both H_o and dH_o/dx will be zero.

19. *Reconstruction of Bending Moments and Centroid Co-ordinates after Deformation*

The solution for σ is sufficient to enable all the geometrical and dynamic features of the deformed blade to be reconstructed.

The curve H can be found from equation (17), para. 16:—

$$H = H_o + \int_r^x (\sigma/\cos^2\phi)\, dx$$

and

$$dH/dx = dH_o/dx + \sigma/\cos^2\phi.$$

The moment of the centrifugal forces is given by equations (15a) and (17).

$$m = \cos\phi \int_l^x C\,(dH_o/dx + \sigma/\cos^2\phi)\,dx$$

and the resultant moment is :—

$$M = m - X,$$

from which stresses at various stations can be calculated.

The centroid co-ordinates are obtained from σ by the relation :—

$$z = z_o + l\int_r^x \sigma dx$$

expressing that σ is in effect the difference before and after strain of the projection on the $\overline{ZX}$ plane of the slope of the locus of centroids.

We then find the lateral deflection :—

$$y = (z - H)/\tan\phi.$$

From the co-ordinates z, y, z_o, y_o, an end elevation of the locus of centroids before and after strain can be drawn, showing how far and in what direction the blade has been deflected.

20. *Initial Tilt*

Where the initial tilt has not been decided upon, but is required to be found as the result of the calculation, it is necessary to divide the solution for σ into two terms, as indicated by the function Q in the second integral of the complete solution.

We have by definition :—

$$Q = \cos^2\phi\ [dH_o/dx - S/C].$$

Let α be a constant angle equal to the tilt, while dH_o/dx is the slope due to initial curvature only; we can write :—

$$Q = \cos^2\phi\ [\alpha + dH_o/dx - S/C]$$

and

$$\sigma = \sigma_1 \int_r^x (b/\sigma_1^2) \int_1^x \sigma_1 \alpha \cos^2\phi\ (dH_o/dx - S/C)\,dx dx$$

$$+ \alpha\sigma_1 \int_r^x (b/\sigma_1^2) \int_1^x \sigma_1 \alpha \cos^2\phi\, dx dx,$$

since α is a constant.

The resultant bending moment will then also appear in the form of two terms such as :—

$$M = M_1\alpha + M_2$$

and α can be chosen to give M any value required to adjust the stresses.

21. *Treatment of Root*

The solution just developed is exact between $x = r$ and $x = l$, where the pitch is constant, so long as the assumption of a solidified root is retained.

Three methods have been outlined for dealing with the root and these will now be examined in detail.

(*a*) *The Root Remains Solidified.*

This is undoubtedly a wide approximation, but the error is on the safe side, and the numerical calculations are extremely simple.

The procedure consists in performing the operations indicated in equation (13), paragraph 14, which is valid for any form of the pitch function, for all stations between $x=r$ and $x=0$, namely :—

$$M/\cos\phi = -\int_1^x (dC/d\xi)\,[\zeta - p\,(\eta/\xi) - H]\,d\xi - X/\cos\phi.$$

For applications the following form is advisable, separating the integrals and isolating the functions of x from the functions of ξ :—

$$M/\cos\phi = \int_1^x (dC/d\xi)\,\zeta d\xi - p\int_1^x (dC/d\xi)\,(\eta/\xi)\,d\xi - HC - X/\cos\phi.$$

The integrals can be carried from 1 to r without a break. Between r and 0 the increments must be added to the values of $\int_1^r$ and the result multiplied by the appropriate value of p before proceeding to the next station. The term HC is a function of x only. In view of the assumption of solidification the values η_0, ζ_0 and H_0 are used between r and 0, but the ordinates of the deformed outer portion, η and ζ, enter into the integrations from 1 to r.

(*b*) *Root Allowed to Deflect so that the Slope at $x=r$ is Continuous.*

By this method it is assumed that dH/dx at $x=r$ consists of the slope present in a state of ease and of a further unknown slope ϵ, due to the deflection of the root.

The addition of ϵ operates as a constant tilt for the outer portion of the blade, and the ordinates η and ζ, and consequently the effective bending moment throughout the blade, will appear in the form (see para. 20) :—

$$\eta_1\epsilon + \eta_2$$
$$\zeta_1\epsilon + \zeta_2$$
$$M_1\epsilon + M_2$$

The moment is then extended from r to 0 by the method used in (*a*) above, and the slope consistent with this moment is calculated between 0 and r by the relations :—

$$dz/dx = dz_0/dx + \int_0^r [(M_1\epsilon + M_2)/EI]\cos\phi$$

$$dy/dx = dy_0/dx - \int_0^r [(M_1\epsilon + M_2)/EI]\sin\phi$$

and

$$(dH/dx)_r = (dz/dx)_r - p_r\,(dy/dx)_r$$

which will take the form :—

$$(dH/dx)_r = i_1\epsilon + i_2.$$

A solution for ϵ then appears by identification of slopes at r, expressed by the equation :—

$$\epsilon + (\partial H_0/\partial x)_r = (i_1)_r\epsilon + (i_2)_r,$$

where $(\partial H_0/\partial x)_r$ is the known slope of H_0 at r in a state of ease.

We then have :—

$$\epsilon = [(i_1)_r - (\partial H_0/\partial x)_r]/[1 - (i_1)_r]$$

to determine ϵ, and consequently the bending moments throughout the blade.

(*c*) *Application by Trial and Error of the General Equation* (12*a*).

This process, as previously stated, is not of practical interest, and is merely indicated here as a possible solution for very abnormally pitched airscrews.

PART IV.—APPLICATIONS

General Remarks

It is a characteristic of impellers that resultant bending moments from fluid reaction can be relieved or partially relieved by moments of opposite hand set up by centrifugal force in the blades. The desired relief is commonly obtained by giving each blade a slight forward tilt from root to tip. Special advantage is taken of this property in the Fairey " D " type of metal airscrew, a comparatively thin, twisted strip of metal, fashioned into an aerofoil cross-section.

The example appended is an engineering application of the theory to the usual accuracy of a ten-inch slide rule. All the essential steps are shown, but well known processes, such as the determination of centroid co-ordinates in a state of ease, are omitted. The blade considered has, when not loaded, a locus of centroids of general form, but application of this method to blades, the centroids of whose sections lie on a mathematical curve, would make the calculation more direct. For any example, however, care in the calculation of centroid co-ordinates and values of $\partial H/\partial x$ in the state of ease is well repaid. The blade of the airscrew considered in the example is twisted about a centre line running from root to tip on the flat under-surface and the origin of co-ordinates is the intersection of this line and the axis of rotation.

To give a value to the constant I_1 it is found convenient to assume the blade extending to its maximum radius without chamfer in the tip region. The curve of moments of inertia is extrapolated as shown in the inset (graph 1), and a value is thus given to I_1. The value of the centrifugal force at the innermost station of the constant pitch portion is taken for C_0. All non-dimensional coefficients are defined in the paper (para. 16) and numerical derivation of their respective values is included in the example.

The limits of constant pitch having been found (graph 2), the calculation proceeds to deal only with this length of blade until the appropriate moments due to centrifugal force, and centroid co-ordinates in the state of strain, are evaluated. Forming an important feature of this process, the construction of the series for σ_1 is worthy of note and a specimen calculation of one of the terms has been included. It is found that this series builds up quickly to the accuracy required once the computer is familiar with the procedure.

Treatment of the root portion is seen to be, to some extent, a matter of choice depending on the accuracy required (Part III, paras. 13 and 21), and the example proceeds on the assumption of a frozen root extending over the last one-fifth of the blade.

EXAMPLE: "FAIREY III.F. PANTHER" METAL AIRSCREW.
CASE: FULL THROTTLE FLIGHT AT 10,000 FT.
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FAIREY III.F. PANTHER AIRSCREW, Drg. No. 56732 A/1X1.

Airscrew Diameter: 11 ft. 9 ins.

TABLE I.

Data for blade cross-sections and pitch angle ϕ.

Relative Radius. x	Breadth of Section. b ins.	Maximum Thickness. m ins.	M. of I. I ins.^{4}	Modulus of Sect. Rear Face. Z_p ins.^{3}	Modulus of Sect. Front Face. Z_s ins.^{3}	Type of Section.	Pitch Angle in degrees ϕ
0	10.50	1.375				Rectangular.	0
.04	9.90	1.375				Intermediate.	15.0
.08	9.36	1.375	1.480			,,	30.0
.12	9.00	1.375	1.455			,,	41.0
.16	8.90	1.375	1.426			,,	47.6
.20	9.05	1.375	1.390	2.26	1.82	,,	49.7
.24	9.30	1.375	1.370			,,	45.5
.28	9.70	1.350	1.300			,,	41.5
.32	10.10	1.310	1.243			,,	37.5
.36	10.65	1.270	1.170			,,	34.3
.40	11.20	1.230	1.064	2.04	1.42	,,	31.6
.44	11.70	1.190	0.950			Aerofoil.	29.3
.48	12.10	1.150	.886			,,	27.3
.52	12.27	1.102	.818			,,	25.4
.56	12.35	1.062	.730			,,	23.7
.60	12.30	1.020	.640	1.54	1.01	,,	22.1
.64	12.20	0.980	.563			,,	21.1
.68	11.95	.940	.490			,,	20.0
.72	11.50	.897	.410			,,	19.0
.76	10.90	.856	.335			,,	18.0
.80	10.10	.816	.270	0.80	0.54	,,	17.1
.84	9.20	.775	.210			,,	16.4
.88	8.10	.720	.143			,,	15.6
.90	7.40	.670	.110	.40	.26	,,	15.3
.92	6.70	.600	.072			,,	15.0
.94	6.00	.500	.040			,,	14.7
.96	4.90	.380	.0135			,,	14.4
.98	3.60	.240	.0028			,,	14.1
1.00	0	0	.0015 = I_1			,,	13.8

TABLE II.

Centroid co-ordinates before strain, and values of H_o and $\partial H_o/\partial x$.
The forward " dihedral " of the blade is 0.75 in. at the tip.

x	y_o ins.	z_o ins.	$y_o \tan \phi$	H_o ins.	$\partial H_o/\partial x$	Faired values $\partial H_o/\partial x$
0	0	.688	0	.688		
.04	−.122	.708	−.033	.741		
.08	−.212	.650	−.122	.772		
.12	−.286	.660	−.249	0.909		
.16	−.333	.675	−.364	1.039		
.20	−.344	.695	−.406	1.101	+.0220	−.0020
.24	−.320	.724	−.315	1.039	−.0220	−.0185
.28	−.259	.767	−.229	0.996	−.0153	−.0170
.32	−.183	.815	−.140	.955	−.0145	−.0146
.36	−.070	.869	−.048	.917	−.0135	−.117
.40	+.082	.938	+.050	.888	−.0103	−.0070
.44	.176	.977	.099	.878	−.0035	0
.48	.217	0.995	.112	.883	+.0018	+.0024
.52	.250	1.010	.119	.891	.0028	.0030
.56	.274	1.020	.120	.900	.0032	.0030
.60	.296	1.024	.120	.904	.0014	.0030
.64	.304	1.029	.117	.912	.0028	.0030
.68	.312	1.036	.114	.922	.0035	.0030
.72	.310	1.038	.107	.931	.0032	.0030
.76	.309	1.042	.101	.941	.0035	.0030
.80	.307	1.045	.094	.951	.0035	.0030
.84	.294	1.046	.087	.959	.0028	.0030
.88	.263	1.035	.073	.960	+.0011	.0030
.90	.238	1.025	.065	.960	0	.0030
.92	.212	1.020	.057	.963	+.0021	.0030
.94	.183	1.015	.048	.967	.0028	.0030
.96	.153	1.010	.038	.972	.0035	.0030
.98	.099	1.005	.025	.975	.0021	.0030
1.00	0	1.000	0	1.000	.0177	.0030

TABLE III.

Values of $b=(I_1/I)\cos^2\phi$. $I_1=.0015$ ins.^{4}

x	I_1/I	$\cos^2\phi$	$(I_1/I)\cos^2\phi$
.20	.00108	.419	.00045
.24	.00110	.491	.00054
.28	.00115	.560	.00065
.32	.00121	.630	.00076
.36	.00128	.682	.00087
.40	.00141	.727	.00103
.44	.00158	.762	.00120
.48	.00170	.789	.00134
.52	.00184	.817	.00150
.56	.00206	.840	.00173
.60	.00235	.860	.00202
.64	.00267	.870	.00232
.68	.00306	.880	.00270
.72	.00366	.894	.00328
.76	.00448	.905	.00405
.80	.00556	.913	.00508
.84	.00715	.920	.00658
.88	.01049	.927	.00973
.90	.01363	.930	.01268
.92	.02085	.933	.01949
.94	0375	.937	.03513
.96	1112	.939	.1043
.98	.5360	.940	.504
1.00	1.000	.942	.942

TABLE IV.

Values of $a=(C/C_o)\,(K_o/\cos^2\phi)$.

x	C lb.	$(C/C_o)\,K_o$	$(C/C_o)\,(K_o/\cos^2\phi)$
.20	$88000=C_o$	28400	67850
.24	85500	27600	56150
.28	82300	26570	47400
.32	78900	25500	40500
.36	75100	24220	35580
.40	70700	22800	31380
.44	66000	21300	27970
.48	61000	19680	24980
.52	56000	18070	22150
.56	50500	16300	19390
.60	45000	14530	16870
.64	39000	12590	14480
.68	32500	10500	11660
.72	27000	8710	9740
.76	21500	6935	7660
.80	16000	5160	5650
.84	10900	3520	3813
.88	6300	2035	2195
.90	4500	1453	1563
.92	3000	968	1031
.94	1800	580	620
.96	850	274	292
.98	410	133	141
1.00	0	0	0

$l=70.5$ ins. $C_o=88000$ $K_o/C_o=.3215$

$K_o=C_o l^2/EI_1=88000\times 4970/10.3\times 10^6\times .0015=28400$

TABLE V.

AIR LOAD BENDING MOMENT AND SHEAR.

$M_T \equiv$ Moment due to thrust. $M_Q \equiv$ Moment due to torque force.

x	$X = M_T \cos\phi + M_Q \sin\phi$	$X/\cos\phi$	$Sldx$	ldx	S (mean) lb.	S (faired)	S/C
.20	18260	28210				1740	.01979
			4390	2.82	1558		
.24	16700	23820				1400	.01637
			3470	2.82	1231		
.28	15250	20350				1140	.01387
			3060	2.82	1087		
.32	13700	17290				970	.01230
			2440	2.82	865		
.36	12250	14850				820	.01093
			2170	2.82	770		
.40	10800	12680				720	.01018
			1910	2.82	677		
.44	9400	10770				640	.00970
			1550	2.82	550		
.48	8200	9220				554	.00909
			1460	2.82	518		
.52	7000	7760				490	.00875
			1320	2.82	468		
.56	5900	6440				440	.00872
			1150	2.82	408		
.60	4900	5290				390	.00867
			1025	2.82	364		
.64	3980	4265				340	.00873
			955	2.82	338		
.68	3100	3310				300	.00923
			880	2.82	312		
.72	2300	2430				250	.00927
			725	2.82	257		
.76	1620	1705				210	.00977
			596	2.82	212		
.80	1060	1109				170	.01063
			420	2.82	149		
.84	660	689				134	.01230
			336	2.82	119		
.88	340	353				112	.01780
			135	1.41	96		
.90	210	218				75	.01665
			94	1.41	67		
.92	120	124				58	.01933
			72	1.41	51		
.94	50	52				40	.0222
			31	1.41	22		
.96	20	21				25	.0295
			11	1.41	8		
.98	10	10				9	.0220
			10	1.41	7		
1.00	0	0				0	0

TABLE VI.

Values of Particular Integral σ_1.

x	σ_1	a	$a\sigma_1$	$a\sigma_1 dx$	$\int a\sigma_1 dx$	b	$b\int a\sigma_1 dx$	$bdx\int a\sigma_1 dx$	$\sigma_1 - 1$
.20	11.317	67850	765000		96428	.00045	43.40		10.310
			1309000	26180			81.30	1.626	
.24	9.6812	56150	544000		70248	.00054	37.90		8.6844
			933000	18660			71.40	1.428	
.28	8.2521	47400	389000		51588	.00065	33.50		7.2564
			672800	13456			62.50	1.2500	
.32	7.0028	40500	283800		38132	.00076	29.00		6.0064
			494500	9890			53.60	1.0720	
.36	5.9321	35580	210700		28242	.00087	24.60		5.9344
			368000	7360			46.00	.9200	
.40	5.0140	31380	157300		20882	.001025	21.40		4.0144
			275300	5506			39.83	.7966	
.44	4.2193	27970	118000		15376	.00120	18.43		3.2178
			206800	4136			33.51	.6702	
.48	3.5509	24980	88800		11240	.00134	15.08		2.5476
			155400	3108			27.28	.5456	
.52	3.0064	22150	66600		8132	.00150	12.20		2.0020
			116000	2320			22.23	.4446	
.56	2.5493	19390	49400		5812	.00173	10.03		1.5482
			86230	1724			18.28	.3656	
.60	2.1835	16870	36830		4088	.00202	8.25		1.1826
			64060	1218			14.90	.2980	
.64	1.8832	14480	27230		2870	.00232	6.65		.88464
			46400	928			11.89	.2378	
.68	1.6457	11660	19170		1941.99	.002695	5.24		.64684
			33370	667.4			9.42	.1884	
.72	1.4577	9740	14200		1274.59	.00328	4.18		.45844
			24230	484.6			7.378	.14756	
.76	1.3103	7660	10030		789.99	.00405	3.198		.31068
			16810	336.20			5.500	.1100	
.80	1.2006	5650	6780		453.79	.00508	2.302		.2006
			11065	221.30			3.833	.07666	
.84	1.1240	3813	4285		232.49	.00658	1.531		.1240
			6644	132.88			2.501	.05002	
.88	1.0742	2195	2359		99.61	.00973	.970		.0742
			4009	40.09			1.725	.01725	
90	1.0569	1563	1650		59.52	.01268	.755		.0569

TABLE VI (*continued*).

x	σ_1	a	$a\sigma_1$	$a\sigma_1 dx$	$\int a\sigma_1 dx$	b	$b\int a\sigma_1 dx$	$bdx\int a\sigma_1 dx$	$\sigma_1 - 1$
			2723	27.23			1.384	.01384	
.92	1.0431	1031	1073		32.29	.01949	.629		.0431
			1712	17.12			1.161	.01161	
.94	1.0315	620	639		15.17	.03513	.532		.0315
			937	9.37			1.136	.01136	
.96	1.0202	292	298		5.80	.1043	.604		.0202
			439	4.39			1.314	.01314	
.98	1.0071	141	141		1.41	.504	.71		.0071
			141	1.41			.71	.0071	
1.00	1.0000	0				.942			0

CALCULATION OF TERMS IN THE SERIES FOR σ_1.

Example for the interval $x = .60$ to $x = .56$, $dx = .04$.

$$a_n b_n (dx^2/4) = 19390 \times .00173 \times .0004 = .01343.$$

$$b_n (dx/2)\left[a_{n-1}\sigma_{n-1}(dx/2) + \int^{n-1} a\sigma_1 dx\right] + \left(b_{n-1}\int^{n-1} a\sigma_1 dx\right)(dx/2) + \sigma_{n-1}$$

$$= .00173 \times .02\ [36830 \times .02 + 4088] + 8.25 \times .02 + 2.18357$$

$$= .0000346 \times 4825 + 2.34857$$

$$= 2.51557$$

	.01343	4088
		736.6
	2.52900	4824.6
	.02035 ($= 1.516 \times .01343$)	
		2.18357
	2.54935	.1650
		2.34857
		.1670
		2.51557

TABLE VII.

Values of $\int_1^x a\sigma_1 \cos^2\phi\ [\partial H_o/\partial x - S/C]\ dx$. $\quad Q = \cos^2\phi\ [\partial H_o/\partial x - S/C]$.

x	$\partial H_o/\partial x$	$a\sigma_1 \cos^2\phi$	$S/C - \partial H_o/\partial x$	$a\sigma_1 Q$	$2 \times$ mean $do.$	$do. \times dx/2$	$\int_1^x a\sigma_1 Q\ dx$
.20	−.0020	321000	.02180	7000			1286
					16310	326	
.24	−.0185	267000	.03487	9310			960
					16050	321	
.28	−.0170	218000	.03087	6740			639
					11545	231	
.32	−.0146	178700	.02690	4805			408
					8055	161	
.36	−.0117	143700	.02263	3250			247
					5210	104	
.40	−.0070	114300	.01718	1960			143
					2833	57	
.44	0	90000	.00970	873			86
					1341	27	
.48	+.0024	70000	.00669	468			59.3
					781	15.6	
.52	.0030	54450	.00575	313			43.7
					551	11.0	
.56	.0030	41500	.00572	238			32.7
					418	8.36	
.60	.0030	31700	.00567	180			24.4
					316	6.32	
.64	.0030	23730	.00573	136			18.1
					241	4.82	
.68	.0030	16870	.00623	105			13.2
					185	3.70	
.72	.0030	12710	.00627	80			9.54
					141	2.82	
.76	.0030	9070	.00677	61			6.72
					108	2.16	
.80	.0030	6200	.00763	47			4.56
					84	1.68	
.84	.0030	3940	.00930	37			2.88
					69	1.38	
.88	.0030	2183	.01480	32			1.50
					53	0.53	
.90	.0030	1535	.01365	21			.97
					37	.37	
.92	.0030	1000	.01633	16			.60
					28	.28	
.94	.0030	599	.0192	12			.32
					19	.19	
.96	0030	280	.0265	7			.13
					10	.10	
.98	.0030	132	.0190	3			.03
					3	.03	
1.00	0030	0	—	0			0

TABLE VIII.

Values of $\sigma=\sigma_1\int_{\text{tip}}^{x}(b/\sigma_1^2)\int_{1}^{x}\sigma_1 a Q dx dx$.

x	σ_1^2	b/σ_1^2	$(b/\sigma_1^2)\int_1^x a\sigma_1 Q dx$	$2\times$mean $do.$	$do.\times(dx/2)$	σ/σ_1	σ
.20	128	.0000035	.00450			0	0
				.01006	.0002012		
.24	93.7	.0000058	.00556			.0002012	.001945
				.01166	.0002332		
.28	68.0	.0000096	.00610			.0004344	.00358
				.01241	.0002482		
.32	49.0	.0000155	.00631			.0006826	.00478
				.01246	.0002492		
.36	35.1	.0000249	.00615			.0009318	.00553
				.01198	.0002396		
.40	25.1	.0000408	.00583			.0011714	.00585
				.01165	.0002330		
.44	17.8	.0000675	.00582			.0014044	.00592
				.01210	.0002420		
.48	12.6	.0001060	.00628			.0016464	.00584
				.01357	.0002714		
.52	9.01	.0001665	.00729			.0019178	.00576
				.01599	.0003198		
.56	6.50	.000266	.00870			.0022376	.00570
				.01900	.0003800		
.60	4.77	.000423	.01030			.0026176	.00571
				.02213	.0004426		
.64	3.54	.000656	.01183			.0030602	.005755
				.02434	.0004868		
.68	2.71	.000944	.01251			.0035470	.00584
				.02726	.0005452		
.72	2.12	.001546	.01475			.0040922	.00596
				.03058	.0006116		
.76	1.72	.002357	.01583			.0047038	.00616
				.03193	.0006386		
.80	1.44	.003527	.01610			.0053424	.00640
				.03116	.0006232		
.84	1.26	.005225	.01506			.0059656	.00671
				.02769	.0005538		
.88	1.155	.008420	.01263			.0065194	.00700
				.02363	.0002363		
.90	1.12	.01132	.01100			.0067557	.00714
				.02173	.0002173		
.92	1.09	.01789	.01073			.0069730	.00726
				.02133	.0002133		
.94	1.06	.03315	.01062			.0071863	.00740
				.0236	.0002360		
.96	1.04	.1003	.0130			.0074223	.00758
				.0280	.0002800		
.98	1.016	.4970	.0150			.0077023	.00776
				.0150	.0001500		
1.00	1.000	.9420	0			.0078523	.00785

TABLE IX.

Values of $m/\cos\phi = \int_{1}^{x} (C/\cos^2\phi)\, s\,dx$. $\qquad s = \sigma + s_0$.

x	$s_0 = (\partial H_0/\partial x)\cos^2\phi$	s	$C/\cos^2\phi$	$Cs/\cos^2\phi$	$(Cs/\cos^2\phi)\,ldx$	$m/\cos\phi$
.20	−.00084	−.00084	210000	−176		3346
					−2000	
.24	−.00908	−.007135	174200	−1243		5346
					−2980	
.28	−.00952	−.00594	147000	−873		8326
					−2010	
.32	−.00919	−.00441	125300	−552		10336
					−1159	
.36	−.00798	−.00245	110100	−270		11495
					− 276	
.40	−.00509	+.00076	97300	+ 74		11771
					+ 825	
.44	0	.00592	86600	512		10946
					1566	
.48	+.00189	.00773	77400	599		9380
					1638	
.52	.002455	.008215	68500	563		7742
					1490	
.56	.00252	.00822	60200	495		6252
					1310	
.60	.00258	.00829	52350	434		4942
					1140	
.64	.00261	.008365	44800	374		3802
					969	
.68	.002645	.008485	36900	313		2833
					809	
.72	.00268	.00864	30200	261		2024
					664	
.76	.002715	.008875	23750	210		1360
					521	
.80	.00274	.00914	17530	160		839
					383	
.84	.00276	.00947	11860	112		456
					252	
.88	.00278	.00978	6800	67		204
					81	
.90	.00279	.00993	4840	48		123
					56	
.92	.00280	.01006	3215	32		67
					36	
.94	.00281	.01021	1923	19		31
					20	
.96	.00282	.01040	905	9		11
					8	
.98	.00282	.01058	436	4		3
					3	
1.00	.002825	.010675	0	0		0

TABLE X.

Values of $\begin{cases} H - H_r = \int_r^x (s/\cos^2\phi)\, l dx. \\ z - z_0 = \int_r^x \sigma l dx. \end{cases}$

x	$s/\cos^2\phi$	2 × mean $do.$	$(s/\cos^2\phi)\, ldx$	$H - H_r$	2 × mean σ	σldx	$z - z_0$
.20	−.00200			0			0
		−.01653	−.02328		.001945	.002740	
.24	−.01453			−.02328			.002740
		−.02515	−.03540		.005525	.007785	
.28	−.01062			−.05868			.010525
		−.01762	−.02480		.00836	.01179	
.32	−.00700			−.08348			.02231
		−.01059	−.01491		.01031	.01453	
.36	−.00359			−.09839			.03684
		−.00255	−.00359		.01138	.01603	
.40	+.001044			−.10198			.05287
		+.00882	+ 01243		.01177	.01659	
.44	.00778			−.08955			.06946
		.01757	.02475		.01176	.01658	
.48	.00979			−.06480			.08604
		.01985	.02796		.01160	.01635	
.52	.01006			−.03684			.10239
		.01985	.02796		.01146	.01613	
.56	.00979			−.00888			.11852
		.01936	.02724		.01141	.01609	
.60	.00957			+.01836			.13461
		.01919	.02702		.011765	.01657	
.64	.00962			.04538			.15118
		.01924	.02710		.011595	.01632	
.68	.00962			.07248			.16750
		.01928	.02716		.01180	.01663	
.72	.00966			.09964			.18413
		.01946	.02740		.01212	.01708	
.76	.00980			.12704			.20121
		.01981	.02790		.01256	.01768	
.80	.01001			.15494			.21889
		.02032	.02860		.01311	.01847	
.84	.01031			.18354			.23736
		.02085	.02940		.01371	.01932	
.88	.01054			.21294			.25668
		.02121	.01496		.01414	.00996	
.90	.01067			.22790			.26664
		.02145	.01513		.01440	.01016	
.92	.01078			.24303			.27680
		.02168	.01528		.01466	.01033	
.94	.01090			.25831			.28713
		.02198	.01549		.01498	.01056	
.96	.01108			.27380			.29769
		.02234	.01573		.01534	.01081	
.98	.01126			.28953			.30850
		.02259	.01592		.01561	.01100	
1.00	.01133			.30545			.31950

TABLE XI.

Values of $y=(z-H)/\tan\phi$.

[$p\,(\eta/\xi)=z-H$ for helical portion]

x	z	H	$z-H$	y	mean $p\,(\eta/\xi)$
.20	.695	1.101	−.406	−.3445	−.379
.24	.727	1.078	−.351	−.3450	−.308
.28	.778	1.042	−.264	−.2987	−.223
.32	.837	1.018	−.181	−.2360	−.139
.36	.906	1.003	−.097	−.1423	−.053
.40	.991	0.999	−.008	−.0130	+.014
.44	1.046	1.011	+.035	+.0623	.040
.48	1.081	1.036	.045	.0872	.047
.52	1.112	1.064	.048	.1013	.048
.56	1.139	1.092	.047	1071	.044
.60	1.159	1.119	.040	.0986	.037
.64	1.180	1.146	.034	.0882	.033
.68	1.204	1.173	.031	.0852	.026
.72	1.222	1.201	.021	.0610	.018
.76	1.243	1.228	.015	.0462	.012
.80	1.264	1.256	+.008	+.0260	+.003
.84	1.283	1.285	−.002	−.0068	−.012
.88	1.292	1.314	−.022	−.0789	−.029
.90	1.292	1.329	−.037	−.1353	−.042
.92	1.297	1.344	−.047	−.1757	−.052
.94	1.302	1.359	−.057	−.2173	−.062
.96	1.308	1.375	−.067	−.2610	−.072
.98	1.314	1.391	−.077	−.3070	−.082
1.00	1.320	1.406	−.086	−.3503	

TABLE XII.

Values of $\int_l^r p\,(\eta/\xi)\,dC$.

$p = xl \tan \phi$ = constant for helical portion = 17.0

x	dC	mean $p\,(\eta/\xi)$	$p\,(\eta/\xi)\,dC$	$\int_l^r p\,(\eta/\xi)\,dC$
.20				−2042
	2500	−.379	−947	
.24				−1095
	3200	−.308	−986	
.28				− 109
	3400	−.223	−758	
.32				+ 649
	3800	−.139	−528	
.36				1177
	4400	−.053	−233	
.40				1410
	4700	+.014	+ 66	
.44				1344
	5000	.040	200	
.48				1144
	5000	.047	235	
.52				909
	5500	.048	264	
.56				645
	5500	.044	242	
.60				403
	6000	.037	222	
.64				+ 181
	6500	.033	214	
.68				− 33
	5500	.026	143	
.72				− 176
	5500	.018	99	
.76				− 275
	5500	.012	66	
.80				− 341
	5100	+.003	+ 15	
.84				− 356
	4600	−.012	− 55	
.88				− 301
	1800	−.029	− 52	
.90				− 249
	1500	−.042	− 63	
.92				− 186
	1200	−.052	− 62	
.94				− 124
	950	−.062	− 59	
.96				− 65
	440	−.072	− 32	
.98				− 33
	410	−.082	− 33	
1.00				0

$$\int_l^r (\eta/\xi)\,dC = -120.$$

TABLE XIII.

Values of $\int_x^{\bar{x}} \zeta dC$.

x	dC	mean ζ	ζdC	$\int_x^{\bar{x}} \zeta dC$
.20				98244
	2500	.711	1775	
.24				96469
	3200	.753	2410	
.28				94059
	3400	.808	2750	
.32				91309
	3800	.872	3310	
.36				87999
	4400	0.949	4210	
.40				83789
	4700	1.019	4790	
.44				78999
	5000	1.064	5315	
.48				73684
	5000	1.097	5485	
.52				68199
	5500	1.126	6190	
.56				62009
	5500	1.149	6310	
.60				55699
	6000	1.169	7010	
64				48689
	6500	1.192	7745	
.68				40944
	5500	1.213	6680	
.72				34264
	5500	1.233	6770	
.76				27494
	5500	1.254	6900	
.80				20594
	5100	1.274	6490	
.84				14104
	4600	1.288	5930	
.88				8174
	1800	1.292	2320	
.90				5854
	1500	1.294	1940	
.92				3914
	1200	1.299	1558	
.94				2356
	950	1.305	1240	
.96				1116
	440	1.311	576	
.98				540
	410	1.317	540	
1.00				0

TABLE XIV.

$$m/\cos\phi = -\int_1^x \zeta dC + p\int_1^r (\eta/\xi)\, dC + (HC)_x.$$

x	dC	$\int_l^r (\eta/\xi)\, dC$	$p = xl\tan\phi$	$p\int_l^r (\eta/\xi)\, dC$	C
0		−120	0	0	94300
	300				
.04		−120	0.7	−84	94000
	900				
.08		−120	3.3	−396	93100
	1300				
.12		−120	7.3	−876	91800
	1600				
.16		−120	12.3	−1475	90200
	2200				
.20		−120	17.0	−2042	88000

x	dC	ζ	mean ζ	$\zeta\, dC$	$\int_l^x \zeta\, dC$
0		.688			102490
	300		.698	210	
.04		.708			102280
	900		.679	611	
.08		.650			101669
	1300		.655	850	
.12		.660			100819
	1600		.668	1068	
.16		.675			99751
	2200		.685	1507	
.20		.695			98244

x	$\int_l^x \zeta\, dC$	$p\int_l^r (\eta/\xi)\, dC$	$(H_oC)_x$	$m/\cos\phi$	m
0	102490	0	64850	37640	37640
.04	102280	− 84	69600	32764	31600
.08	101669	− 396	71850	30215	26180
.12	100819	− 876	82700	18995	14330
.16	99751	−1475	93650	7576	5100
.20	98244	−2042	96800	3486	2267

TABLE XV.

Summary of Bending Moments.

$$M = X - m.$$

x	X	m	M
0	21600	37640	−16040
.04	21400	31600	−10200
.08	22700	26180	− 3480
.12	21300	14330	+ 6970
.16	19750	5100	14650
.20	18260	2267	15993
.24	16700	3740	12960
.28	15250	6240	9010
.32	13700	8190	5510
.36	12250	9480	2770
.40	10800	10010	+ 790
.44	9400	9545	− 145
.48	8200	8330	− 130
.52	7000	6990	+ 10
.56	5900	5730	170
.60	4900	4570	330
.64	3980	3543	437
.68	3100	2660	440
.72	2300	1915	385
.76	1620	1293	327
.80	1060	801	259
.84	660	437	223
.88	340	197	143
.90	210	119	91
.92	120	65	55
.94	50	30	20
.96	20	11	9
.98	10	3	7
1.00	0	0	0

TABLE XVI.

SUMMARY OF CENTROID CO-ORDINATES.

	State of Ease.		Strained State.	
x	y_o''	z_o''	y''	z''
0	0	.688	0	.688
.04	−.122	.708	−.122	.708
.08	−.212	.650	−.212	.650
.12	−.286	.660	−.286	.660
.16	−.333	.675	−.333	.675
.20	−.344	.695	−.344	.695
.24	−.310	.724	−.345	.727
.28	−.259	.767	−.299	.778
.32	−.183	.815	−.236	.837
.36	−.070	.869	−.142	.906
.40	+.082	.938	−.013	0.991
.44	.176	.977	+.062	1.046
.48	.217	0.995	.087	1.081
.52	.250	1.010	.101	1.112
.56	.274	1.020	.107	1.139
.60	.296	1.024	.099	1.159
.64	.304	1.029	.088	1.180
.68	.312	1.036	.085	1.204
.72	.310	1.038	.061	1.222
.76	.309	1.042	.046	1.243
.80	.307	1.045	+.026	1.264
.84	.294	1.046	−.007	1.283
.88	.263	1.035	−.079	1.292
.90	.238	1.025	−.135	1.292
.92	.212	1.020	−.176	1.297
.94	.183	1.015	−.217	1.302
.96	.153	1.010	−.261	1.308
.98	.099	1.005	−.307	1.314
1.00	0	1.000	−.350	1.320

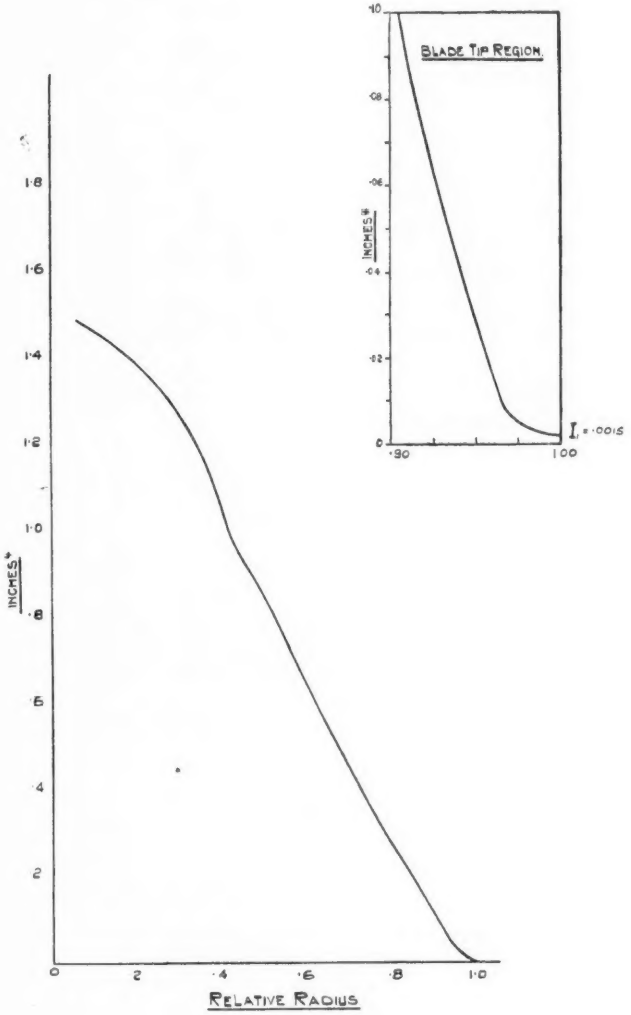
TABLE XVII.

Summary of Stresses.

Full Throttle Flight at 10,000 ft.

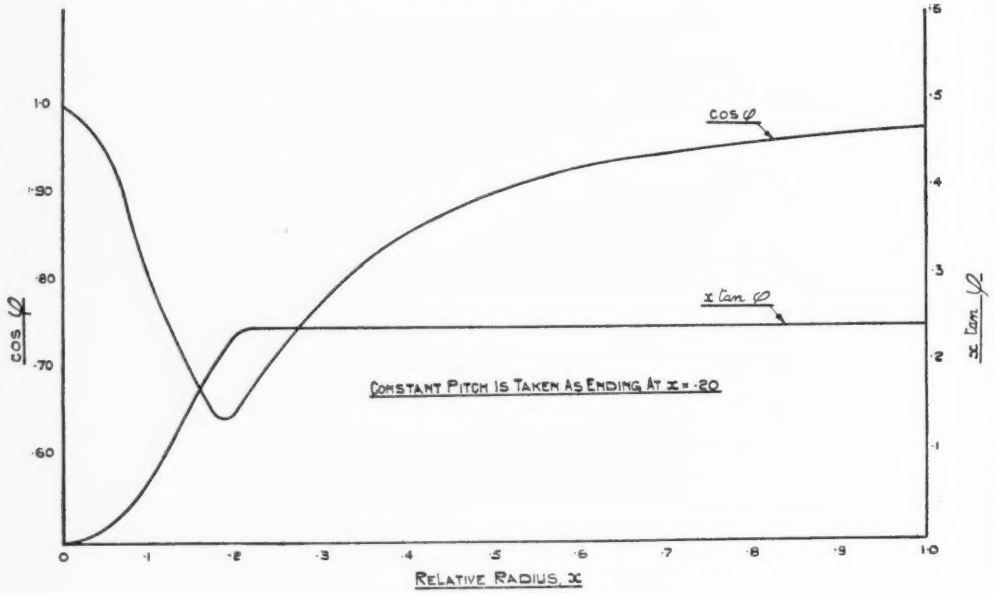
Section	XX	AA	BB	CC	DD	EE	FF	GG	HH	JJ	KK
Radius ft.	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5
Area A ins.^{2}	11.30	10.58	10.4	10.4	9.69	9.54	8.84	7.83	6.40	4.77	2.31
Z comp. ins.^{3}	1.977	1.872	1.718	1.640	1.329	1.212	1.035	0.839	.620	.415	.142
Z tension ins.^{3}	2.351	2.284	2.230	2.210	1.993	1.818	1.553	1.259	0.930	.623	.212
C.F. lb.	93030	89650	84380	77020	67850	57200	45120	32480	20250	9520	2000
Net B.M. lb. ins.	−2000	+15500	11000	4000	0	0	300	400	350	200	100
C/A lb./ins.^{2}	8230	8480	8120	7405	7000	6000	5105	4150	3165	1995	867
M/Z tens. lb./ins.^{2}	−851	+6790	4930	1813	0	0	193	318	377	321	472
M/Z comp. lb./ins.^{2}	−1013	+8280	6410	2443	0	0	290	477	565	482	705
Total stress tension	7379	15270	13050	9218	7000	6000	5298	4468	3542	2316	1339
Total stress tons/ins.^{2}	3.30	6.82	5.82	4.11	3.13	2.68	2.36	2.00	1.58	1.03	0.60
Total stress comp.	−9243	−200	−1710	−4962	−7000	−6000	−4815	−3673	−2600	−1513	−162
Total stress tons/ins.^{2}	−4.12	−0.09	−0.78	−2.22	−3.13	−2.68	−2.15	−1.64	−1.16	−0.68	−0.07

Fairey III.F. Panther Airscrew.
Moments of Inertia of Blade Cross Sections.



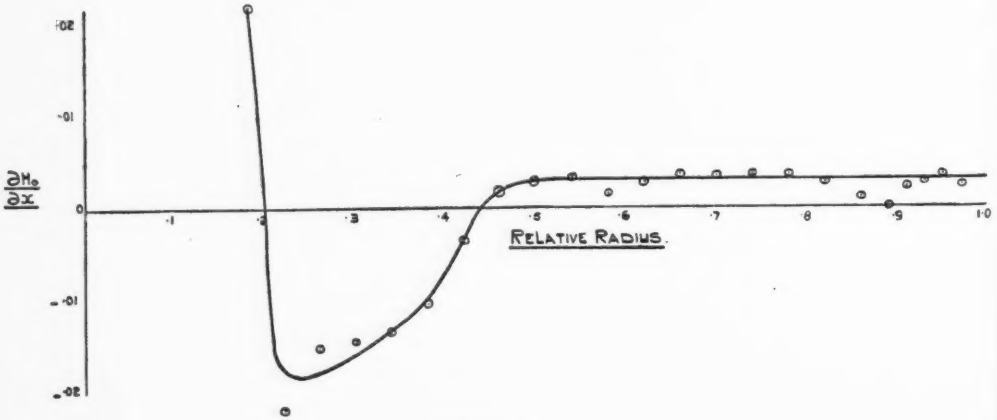
GRAPH 1.

Fairey III.F. Panther Airscrew.
Pitch Distribution along the Blade.



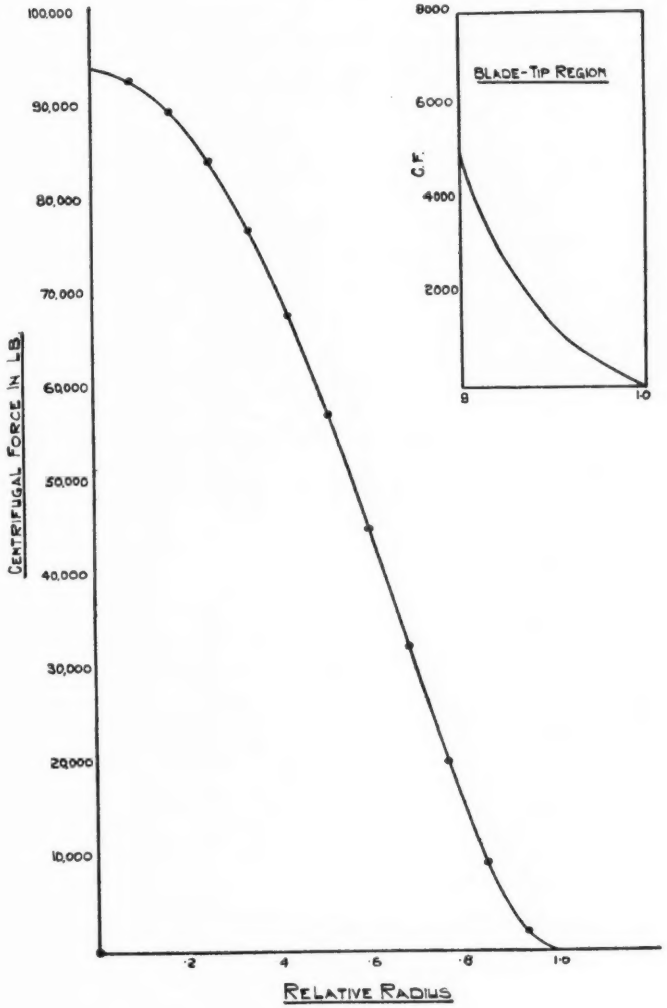
Graph 2.

Fairey III.F. Panther Airscrew.
Values of $\partial H_o/\partial x$.

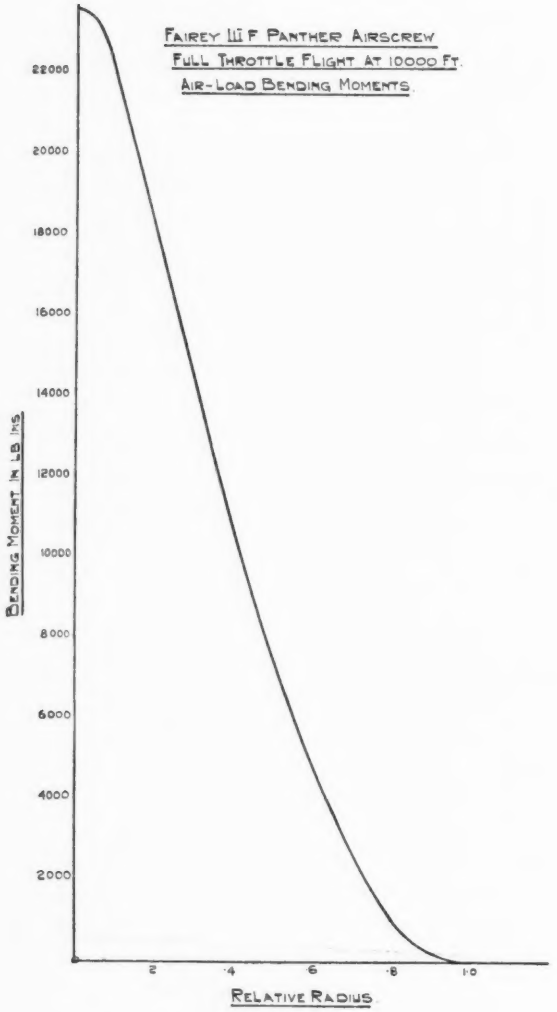


Graph 3.

Fairey III.F. Panther Airscrew.
Full Throttle Flight at 10,000 ft.
Distribution of Centrifugal Force.

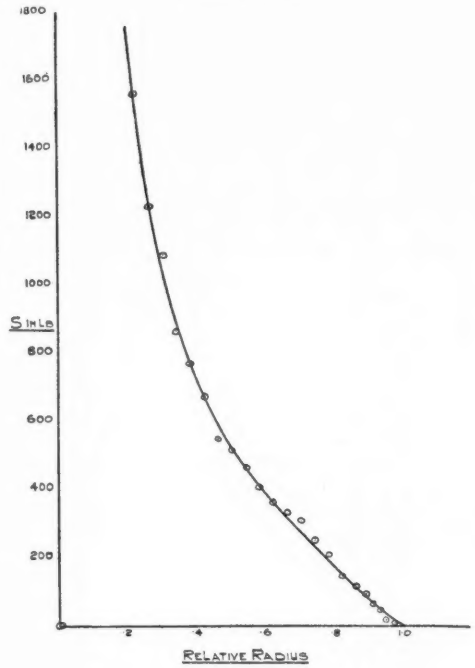


GRAPH 4.


FAIREY III F PANTHER AIRSCREW
FULL THROTTLE FLIGHT AT 10000 FT.
AIR-LOAD BENDING MOMENTS.
BENDING MOMENT IN LB INS.
22000
20000
18000
16000
14000
12000
10000
8000
6000
4000
2000
·2
·4
·6
·8
1·0
RELATIVE RADIUS

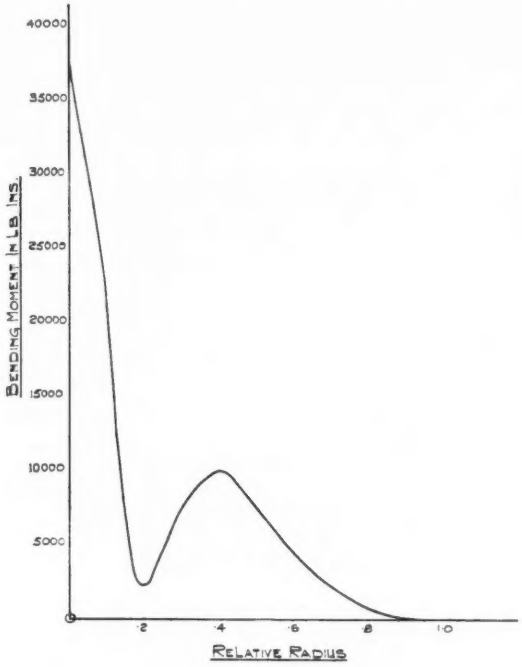
GRAPH 5.

Fairey III.F. Panther Airscrew.
Full Throttle Flight at 10,000 *ft.*
Values of S.



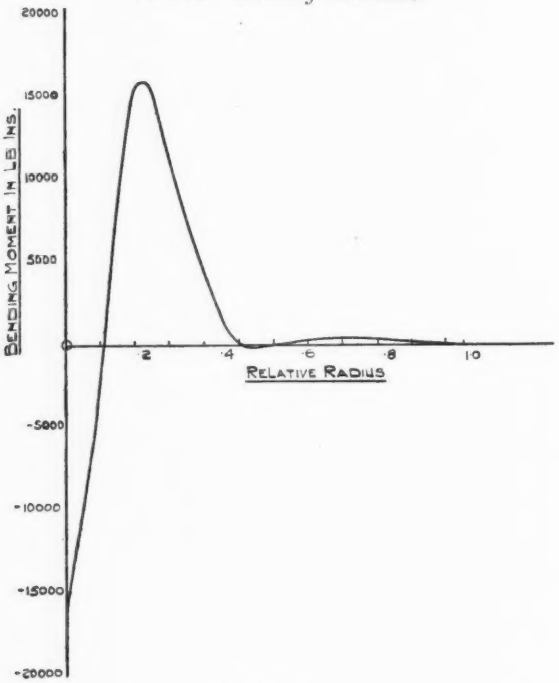
GRAPH 6.

Fairey III.F. Panther Airscrew.
Full Throttle Flight at 10,000 *ft.*
Bending Moments due to Centrifugal Force.



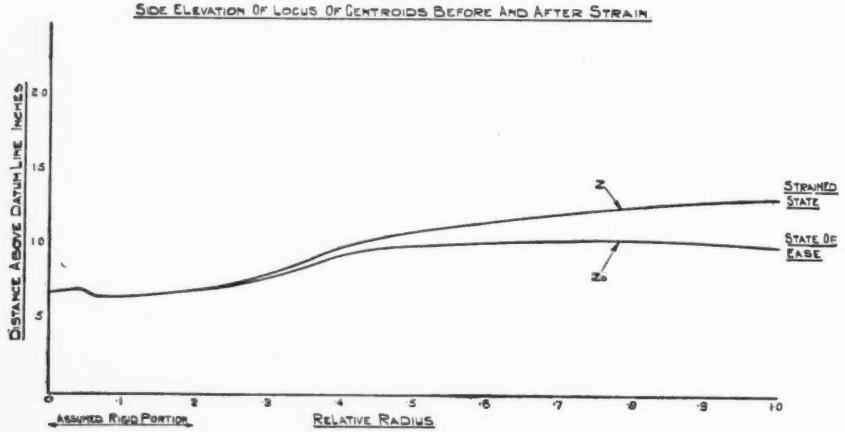
GRAPH 7.

Fairey III.F. Panther Airscrew.
Full Throttle Flight at 10,000 ft.
Resultant Bending Moments.

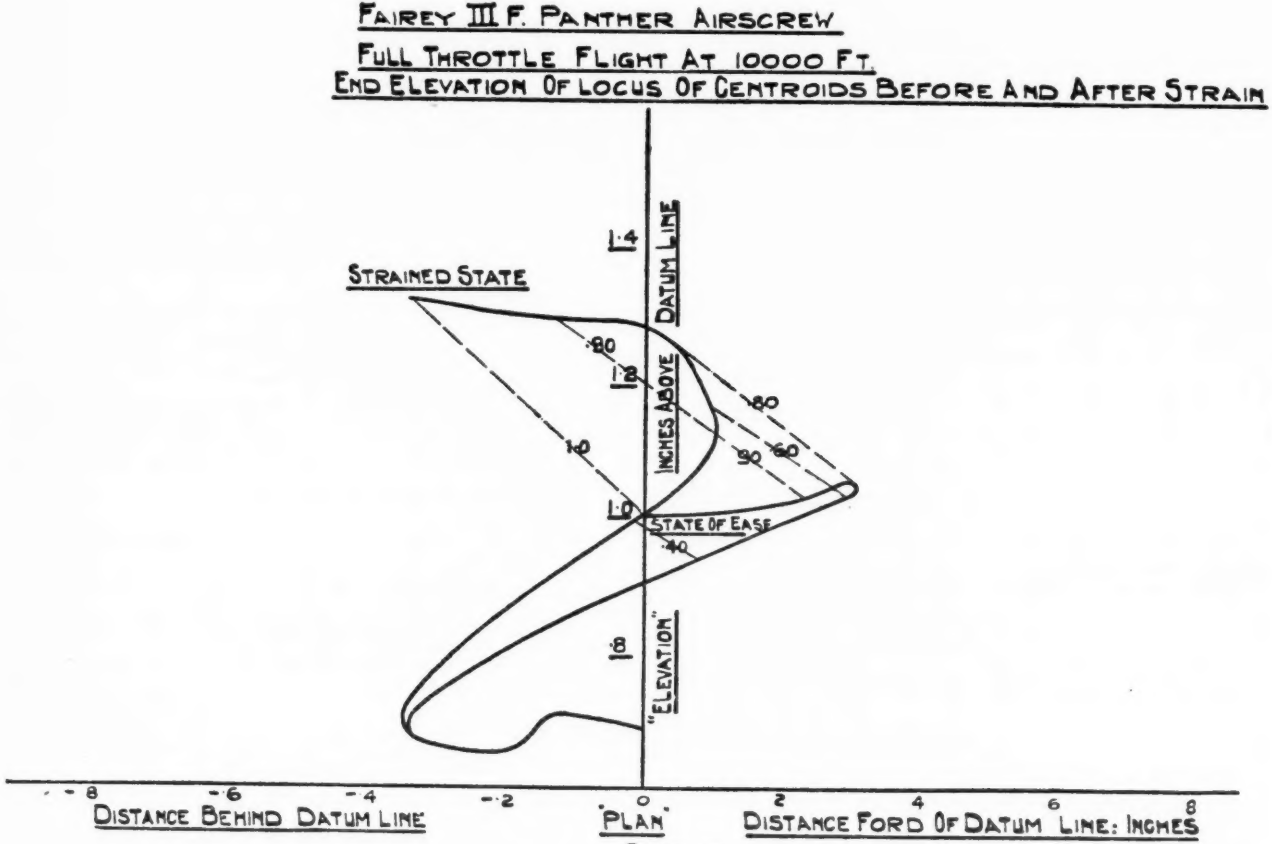


GRAPH 8.

FAIREY III F PANTHER AIRSCREW
FULL THROTTLE FLIGHT AT 10000 FT
SIDE ELEVATION OF LOCUS OF CENTROIDS BEFORE AND AFTER STRAIN

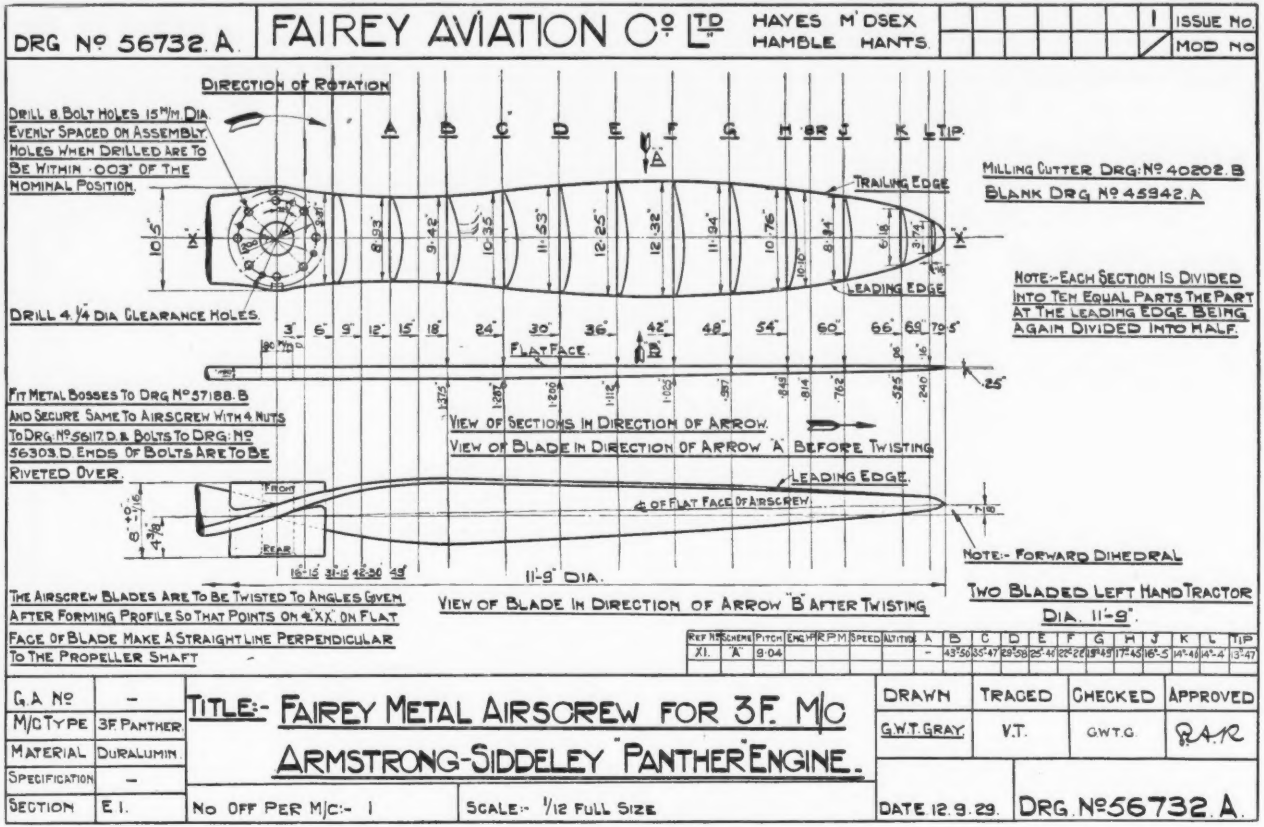


GRAPH 9.



GRAPH 10.

(Figures on Broken Lines Show Relative Radius.)


DRG Nº 56732.A.
FAIREY AVIATION Cº LTD
HAYES M'DSEX
HAMBLE HANTS.
ISSUE NO.
MOD NO
DIRECTION OF ROTATION
DRILL 8 BOLT HOLES 15 M/M DIA EVENLY SPACED ON ASSEMBLY. HOLES WHEN DRILLED ARE TO BE WITHIN .003" OF THE NOMINAL POSITION.
A B C D E F G H BR J K L TIP
TRAILING EDGE
LEADING EDGE
MILLING CUTTER DRG: Nº 40202.B
BLANK DRG Nº 45942.A
NOTE:-EACH SECTION IS DIVIDED INTO TEN EQUAL PARTS THE PART AT THE LEADING EDGE BEING AGAIN DIVIDED INTO HALF.
DRILL 4 1/4 DIA CLEARANCE HOLES.
FLAT FACE
FIT METAL BOSSES TO DRG Nº 57188.B AND SECURE SAME TO AIRSCREW WITH 4 NUTS TO DRG: Nº 56117.D & BOLTS TO DRG: Nº 56303.D ENDS OF BOLTS ARE TO BE RIVETED OVER.
VIEW OF SECTIONS IN DIRECTION OF ARROW.
VIEW OF BLADE IN DIRECTION OF ARROW 'A' BEFORE TWISTING
LEADING EDGE.
℄ OF FLAT FACE OF AIRSCREW.
FRONT
REAR
NOTE:- FORWARD DIHEDRAL
11'-9" DIA.
THE AIRSCREW BLADES ARE TO BE TWISTED TO ANGLES GIVEN AFTER FORMING PROFILE SO THAT POINTS ON ℄ 'XX' ON FLAT FACE OF BLADE MAKE A STRAIGHT LINE PERPENDICULAR TO THE PROPELLER SHAFT
VIEW OF BLADE IN DIRECTION OF ARROW "B" AFTER TWISTING
TWO BLADED LEFT HAND TRACTOR DIA. 11'-9".
REF Nº | SCHEME | PITCH | ENG H.P. | R.P.M. | SPEED | ALTITUDE | A | B | C | D | E | F | G | H | J | K | L | TIP
XI. | 'A' | 9·04 | - | 43'-50 | 35'-47 | 29'-58 | 25'-46 | 22'-28 | 19'-48 | 17'-45 | 16'-5 | 14'-48 | 14'-4 | 13'-47
G.A. Nº | -
M/C TYPE | 3F PANTHER.
MATERIAL | DURALUMIN.
SPECIFICATION | -
SECTION | E 1.
TITLE:- FAIREY METAL AIRSCREW FOR 3F. M/C ARMSTRONG-SIDDELEY "PANTHER" ENGINE.
No OFF PER M/C:- 1
SCALE:- 1/12 FULL SIZE
DRAWN | TRACED | CHECKED | APPROVED
G.W.T.GRAY | V.T. | GW.T.G. | R.A.R
DATE 12.9.29.
DRG. Nº 56732.A.

GRAPH II.

TABLES FOR USE IN AERO NAVIGATION BY ASTRONOMICAL METHODS

BY

P. F. EVERITT, B.Sc., F.INST.P., F.R.AE.S.

As the subject of the reduction of astronomical observations in the air had been the subject of several conversations with the late Squadron Leader Johnston, who was one of our best exponents of aerial navigation, Mr. A. J. Hughes, O.B.E., suggested to the writer that a short note based on Squadron Leader Johnston's methods would be useful in order that his valuable experience should not be entirely lost, and that some systematic survey of the methods available would be useful in connection with a subject that is becoming of increasing importance now that the range and reliability of aircraft have been so largely extended.

Squadron Leader Johnston was the navigator on both the R.100 and R.101, and his methods were worked out for aircraft with enclosed cabin accommodation.* His methods were based on the use of existing navigation tables, and in this note the subject will be restricted to the use of existing tables; the slide rule will not be discussed, nor will any comparison of slide rule and tables be attempted.

Squadron Leader Johnston had collected from various sources the tables that appeared most convenient to him, and had them bound up in one small volume. These were as follows:—

The Alpha, Beta, Gamma Navigation Tables. By H. B. Goodwin, M.A., F.R.A.S. Published by J. D. Potter, London, 1926.

A, B and C Azimuth Tables. Taken from "A Complete Set of Nautical Tables," by J. W. Norie. Published by Imray, Laurie, Norie and Wilson, Ltd.

Parallax and Refraction Tables combined for the Sun.

Parallax and Refraction Tables combined for the Moon.

Refraction Tables for Stars.

These must not have the correction for dip embodied as a bubble sextant was employed, and were taken from various sources not specified.

Tables of Arc to Time and Time to Arc.

Before discussing these tables, the following additional tables were examined:—

Inman's Nautical Tables. Published by J. D. Potter.

Norie's Nautical Tables. Published by Imray, Laurie, Norie and Wilson.

Altitude and Azimuth Tables. By Commdr. Radler de Aquino, Brazilian Navy. Published by J. D. Potter.

Navigation Tables for Mariners and Aviators. By Dreisonstok. Published by Hydrographic Office, U.S. Navy.

Line of Position Book. By Lieut. Comm. P. V. H. Weems, U.S. Navy. Published by U.S. Naval Institute.

Burdwood's Tables (Davis 2). Published by J. D. Potter.

Davis's Star Azimuth Tables (Davis X). Published by J. D. Potter.

* It is to be noted that this enclosed cabin accommodation is now almost universal in all aircraft where astronomical navigation is likely to be employed.

The problem to be dealt with is to find the computed altitude and azimuth of a heavenly body as it would be seen from a given point on the earth's surface at a specified instant of time, so that by comparison with the actual altitude observed at that instant, a position line may be drawn on the chart such that the observer was actually somewhere on that line when the observation was made.

Considering the fundamental spherical triangle *PZS* where *P* is the pole, *Z* the zenith, and *S* the heavenly body, the sides *PZ* and *PS* are known, being the terrestrial co-latitude, or complement of terrestrial latitude, and the polar distance of the heavenly body which is the complement of its declination.

The angle *ZPS* is called the hour angle and represents the interval of time which the heavenly body is distant from the meridian, and is found by combining the terrestrial longitude with the Greenwich time of the observation.

Thus, in the problem we are dealing with, two sides and the enclosed angle are known, and consequently the triangle is completely determined. We require to find the side *ZS*, the zenith distance, or its complement, the altitude, and the angle *PZS* the azimuth.

In the reduction of an observation, there are three parts, first the reduction of the observation from observed altitude to true altitude by applying the necessary corrections for refraction and parallax (dip is excluded when using a bubble sextant), which are all tabulated; secondly, the calculation of the altitude and azimuth as it would have been observed if the observer were at some specified position, and finally by comparison of the calculated and observed results to draw the position line on the chart.

For the calculation of the altitude and azimuth we already know the side *PS* the complement of the declination of the body and the divisions " *a* " and " *b* into which the writer has divided the various tables differ in their treatment of the side *ZP* and the included angle *ZPS*.

On examining the tables mentioned and excluding the classical haversine method which it was desired to replace, it is at once seen that the altitude tables fall into two divisions :—

" *a*." Those in which the tables are arranged to give the altitude for any position.

" *b*." Those in which the tables are arranged to give the altitude for a limited number of positions only, these positions being spaced at intervals in both latitude and longitude.

In the " *a* " division these quantities are taken directly from the dead reckoning, the side *ZP* being the co-altitude and the angle *ZPS* the local hour angle, which is found by combining the dead reckoning longitude with the Greenwich time of the observation.

In the " *b* " division an arbitrary latitude and longitude are assumed, such that, when the latter is combined with the Greenwich time, it will give an hour angle which can be used to enter the tables.

By this artifice of an " assumed position " differing from the dead reckoning position by a known amount, the volume of the tables is reduced and they can be prepared in a form which considerably reduces the amount of work to be done.

When two position lines are worked from simultaneous observations, their intersection gives the absolute position, but when only one observation can be made at any one time such as an observation of the sun, when no moon is available, the most probable point on the single position line is the foot of the perpendicular drawn from the dead reckoning position which, when using the " *b* " class of tables must also be drawn on the chart, but when using the " *a* " class of tables has already been drawn in constructing the position line.

The various tables are accordingly divided as follows:—

"*a*."	"*b*."
Goodwin's Alpha, Beta, Gamma Tables.	Aquino's Altitude and Azimuth Tables.
Aquino's New Altitude Tables.	Dreisonstok's Tables.
	Weem's Line of Position Book.
	Smart and Shearme's Tables in Inman.
	Ogura's Table in Norie's Tables.

Of the above tables only Aquino's Altitude and Azimuth Tables and Dreisonstok's Tables give the azimuth as well as the altitude, so that a separate table must be used for the azimuth.

A priori the tables in division "*a*" are to be preferred if they are compact and convenient, as they avoid the complication of the "assumed position," so they will be examined first.

In order to compare the convenience of the tables a standard example will be worked, assuming lat. 33° N. hour angle 16° E. and declination 21° 14′.2 S. The number of operations and times of entering the tables will be noted and compared. The preliminary work of finding the hour angle is common to all and will not be considered.

By Goodwin's Alpha, Beta, Gamma Tables

	20	
	11.4119	entering Table II with hour angle 16° = 1h. 4m. by subtraction.
Lat. 33° N.	8.5881	
Decl. 21° 14′.2 S.	9.9236	entering Table I with latitude to find *B*.
L − *D* 54° 14′.2.	9.9695	entering Table I with declination to find *B*.
	8.4812	sum of three.
	0′ 30″.2	entering Table I in *B* col. to find corresp.
	9′ 44″.4	entering Table I with *L* − *D* to find difference.
	9′ 14″.2	
Zen. Dist.	56° 20′.7	entering Table I with — to find *ZD*.
Altitude	33° 39′.3	complement of *ZD*.

Total, six table entries and five additions or subtractions.

Precepts.—One only referring to the formation of *L* − *D*.

Goodwin's tables suffer, however, from the serious defect that they only cover hour angles up to eight hours from the meridian, a total of 16 hours out of 24. This defect can in many cases be avoided by suitable choice of stars to be observed and was doubtless so avoided by Squadron Leader Johnston; its existence cannot be passed over and for that reason they cannot be recommended for general use.

*By Aquino's New Altitude Tables**

Hour angle 344° W.	85644 + from *HA* Table.
Lat. 33° N.	3820 + from Lat. or Decl. Table.
Decl. 21° 14′.2 S.	1527 + from Lat. or Decl. Table.
	90991 sum.

* These are not the Aquino altitude and azimuth tables, but are included at the end of the same volume in the 1918 edition.

$L=33°$ N.
$D=21°\ 14'.2$ S.
$L+D=54°\ 14'.2$.

3027 + by entering HA column with the above sum and taking out the corresponding quantity in the sum or diff. column.
41556 + entering with $L+D$ in sum or diff. column.

44583 sum.
Alt. $33°\ 21'$ by entering the sum or diff. col. and reading off from below.

Total, six table entries and three additions.

Two Precepts, one for formation of $L+D$ and the other always to use hour angle West $0°$ to $360°$.

The arrangement of the tables is not very convenient.

Proceeding to the tables in the " b " division.

By Aquino's Altitude and Azimuth Tables

Entering from below with decl. $D=21°\ 0'$ and hour angle $t=16°$ we find an approximate $a=15°$ and approximate $b=22°$ and turn to the page and column headed $a=15°\ 0'$.

Entering with $a=15°\ 0'$ and $b=22°$ we find $d=21°\ 13'$ with $60'=1.05$; multiplying 1.05 by 1.2, the difference between $21°\ 13'$ and $21°\ 14'.2$ the actual declinations we get $1'.2$ or $1'$ to add to the $22°$ of b giving $b=22°\ 1'$; the corresponding entry in the t column is $16°\ 7'$ with $60'=12$; multiplying .12 by $1'.2$ we get $.14'$ to add to the $16°\ 7'$ of t which in this case may be neglected. Thus, starting from $t=16°$ we have shifted it to $t=16°\ 7'$ in order to use the tables, and this involves the use of an assumed longitude differing from the dead reckoning longitude by $7'$.

Going back to $b=22°\ 1'$ we have to form $C=L+b$ modifying L so that C is a whole number of degrees and introducing an assumed latitude to do so; thus, " C "$=55°$ by making the assumed latitude $32°\ 59'$ instead of $33°$.

Re-entering from below with " C "$=55°$ in column N we have the altitude $N=33°\ 39'$ for an assumed longitude differing by $7'$ and an assumed latitude differing by $1'$ from our original dead reckoning position. In column Z is also the azimuth $18°\ 7'$ which in this case will be S. $18°\ 7'$ E.

Total, five table entries on two book openings only, two interpolations and two separate sums to find the assumed latitude and longitude that must be adopted.

Precepts.—There are four of these above for the formation of C and four more with regard to Z.

Owing to the multiplicity of the precepts, and the necessary transference of latitude and longitude, the use of these tables requires special care; in the writer's opinion they are not so suitable for use in the air as are some of the other tables.

By Dreisonstok's Tables

In these tables the assumed position is adopted before entering the tables, and is chosen to be the nearest whole degree in latitude and hour angle to the dead reckoning latitude and hour angle found from the dead reckoning.

Entering Table I with hour angle 16° and latitude 33° N, we write down:—

$b=55°\ 57'.5$ N.	$A=\ 1193$	$C=636$	$Z'=81°.1$
$d=21°\ 14'.2$ S.	$B=24444$	$D=159$	
alg. sum $34°\ 43'.3$	$A+B=25637$	$C+D=795$	$Z''=80°.9$
	Alt. $33°\ 39'$		alg. sum $Z=N\ 162°.0$ E.

The values of b, AC and Z' are found in one line; now combine the value of b which takes its name from the latitude with declination by adding them algebraically and using this quantity as argument, enter Table II and take out B and D which are also given in the same line.

Forming $A+B$ and $C+D$ we enter Table II again backwards and take out the altitude $N°$ and Z''.

By adding Z'' to Z' algebraically we get Z, the azimuth reckoned from the N point.

The table entries are:—

One for four quantities b, A, C, Z'.
One for two quantities B, D.
One for altitude.
One for Z''.

A total of four table entries, and there are also four addition sums.

The precepts to be observed can be reduced to four in number and are very easy ones.

By Weems' Line of Position Book

The procedure is very similar to Dreisonstok without the azimuth part of the table, except that the auxiliary angle used by Weems is the complement of that used by Dreisonstok.

The assumed position is adopted before entering the table and using our standard example we get as follows:—

Entering Table A with hour angle 16° and latitude 33° N. we get K and A.

$K=34°\ 2'.5$ N.	$A=\ 1193$
$d=21°\ 14'.2$ S.	$B=24444$
$55°\ 16'.7$	$A+B=25637$ Altitude $33°\ 39'$

Combining K with d by precept and using their sum to enter Table B we find B and form $A+B$, which is used to enter Table B backwards, giving the altitude.

The table entries are:—

One for two quantities.
Two for one quantity each.

A total of three table entries with two addition sums.

There is only one precept to be observed.

Smart & Shearme's Tables (Inman)

These are really the same tables as the altitude part of Dreisonstok only differently arranged with the further disadvantage of having to refer to a table of log sines as Table II is not given.

Ogura's Tables

These are the original from which Weems' tables are constructed, but they are not so convenient, as the second part, the table of log secants, is not given, these having to be looked out elsewhere.

Azimuth Determination

With regard to the method of determining azimuths, when using tables for the altitude which do not also give the azimuth, there is little doubt that the *A*, *B* and *C* Azimuth Tables (Norie) are excellent, having a very small bulk and covering all possible cases up to 68° declination or latitude in only 40 pages.

Working on the standard example we have:—

$A = 2.26$ S.
$B = 1.41$ S.

$A + B = 3.67$ S. whence $Az = S\ 18°.0$ E.

using three table entries and one addition.

The precepts are very simple and are printed on each page of the tables.

The usual marine practice is to use both Burdwood's Tables covering 0° to 24° declination and 0° to 64° latitude, together with Davis's Star Azimuth Tables covering declinations 23° to 64° and latitudes 0° to 60°.

This means carrying two fairly large volumes, which is open to objection for aviation work, although the result can be obtained by inspection.

The use of the *A*, *B*, *C* Azimuth Tables is so simple and they occupy so little space that they are to be clearly preferred.

On balancing up the advantages and disadvantages of the various tables, the writer considers that the aero-navigator will at present be best served by using Weems' Position Line Book for altitudes, together with the *A*, *B*, *C* Azimuth Tables in Norie; the latter could be taken out and rebound either separately or with the Weems Tables to form either two or one thin books which will contain all he requires both for position line work or compass error observations. The convenience of use is very great, the precepts to be remembered are a minimum, and the necessary tables of refraction, parallax, etc., are already printed in the Weems Book.

Although it is usually best to use the Calculated Altitude and Azimuth to draw the intercept and position line, this can be done without using the azimuth, using instead the intercepts calculated for two suitably chosen assumed positions as proposed by Aquino.

In this case the intercepts are used as radii to draw circles and the position line will then be one of the four tangents common to both circles. Which tangent is to be drawn is at once obvious from the knowledge of the hour angles and whether the position line lies towards or away from the celestial body.

A considerable amount of work in connection with the hour angle is saved by carrying two second setting watches rated to mean time for the sun and sidereal time for the stars which are checked every day. The sun watch, although rated to mean time, is *set daily* to solar apparent time at Greenwich, thus eliminating the equation of time from the calculation.

The star watch, showing sidereal time at Greenwich, needs only to have the R.A. of the star applied to it to give the Greenwich hour angle; by applying the longitude the local hour angle is at once found, without reference to the R.A.M.S. This system has been proposed by Commr. Weems, of the U.S. Navy.

In conclusion, the writer hopes that this note will be of service to those interested in aero-navigation by saving them trouble in searching out suitable tables for use in the air, and also, even though in a very small way, help to preserve the memory of a pioneer in aero-navigation—Squadron Leader Johnston.

VISIT OF THE SECRETARY TO CANADA AND THE UNITED STATES OF AMERICA, JUNE—JULY, 1930

In June and July, 1930, the Secretary paid a visit to Canada and the United States of America. The primary objects of the visit were to bring about a closer and more personal contact with Canadian and American members of the Society, to explore the possibilities of forming Branches of the Society in Canada, and to arrange for exchange of technical information between Canada, the United States of America and this country.

In Canada the Secretary placed himself in the hands of Group-Captain E. W. Stedman, the Chief Aeronautical Engineer of the Department of National Defence, a Fellow of the Society. To the constant and unfailing help given by Group-Captain Stedman the greater part of the success of the visit is due. Under his guidance the Secretary was able to visit Government Departments, to see the work that was being done, and to come into personal contact with the leading aeronautical authorities in Canada. In Ottawa the Secretary met the following members of the Society:—Flt.-Lieut. A. O. Adams, A.M.I.Ae.E. Flt.-Lieut. R. Collis, A.M.I.Ae.E.; Flt.-Lieut. A. Ferrier, B.Sc., A.F.R.Ae.S.; Pilot-Officer A. L. Jones, B.Sc., A.M.I.Ae.E.; Sq.-Ldr. D. Lucking, A.F.R.Ae.S.,; Professor J. H. Parkin, F.R.Ae.S.; Captain Windsor, Companion, and also Dr. R. W. Boyle, M.A., Ph.D., Director of the Division of Pure and Applied Physics, Natural Research Laboratories; Wing-Comdr. L. R. Breadner, A.F.C., Director of Military Branch of the R.C.A.F.; Wing.-Comdr. Cowley, Deputy Director of Civil Aviation; Sq.-Comdr. Forbes; Flt.-Lieut. Grant, in charge of engines, Department of National Defence; Major-General J. H. MacBrien, formerly chief of staff of R.C.A.F.; F. H. Peters, Esq., Director of the Topographical Section, Department of the Interior; Sq.-Ldr. Tudhope; Flt.-Officer van Vleit; J. A. Wilson, Esq., Controller of Civil Aviation, Department of National Defence, and many others.

Everywhere the Secretary found that the name of the Royal Aeronautical Society was not only an open sesame but one which ensured a welcome and a hospitality which was almost embarrassing at times. In Ottawa the Secretary had the opporunity of attending a flying meeting and examining a large number of Canadian and American light planes and of visiting the works of the Ottawa Car Company, which controls Avro aircraft and Armstrong Siddeley engines produced in Canada; the new aeronautical research laboratories in charge of Professor Parkin, which have become the most important centre of research for Canadian aviation; various aerodromes; and the photographic, topographical and other departments where the results of the survey work are co-ordinated and maps prepared. The photographic and topographical sections are highly organised and efficient and undoubtedly the survey work being carried out in Canada is an example to the world of the efficient use of aircraft.

On June 10th the Secretary returned to Montreal till the 15th and discussed at length with Group-Captain Stedman, Mr. R. J. Durley, Secretary of the Engineering Institute of Canada, and members of Council of the Institute, the possibility of forming branches in Canada. As a result of these conversations an agreement was drawn out and later ratified by the Councils of the Institute and the Society. That this agreement should have been found possible is largely due to the untiring energy and help of Group-Captain Stedman, Mr. R. J. Durley and the members of the E.I.C. Council, who gave ungrudgingly of their time to

meet the point of view of the Society. The agreement is given at the end of this note. One of the most important clauses in it ensures a circulation to all members of the Society of information on Canadian technical progress at regular intervals, and of course, a corresponding circulation, by means of the Journal, of British progress.

While in Montreal the Secretary paid a visit to the St. Hubert Aerodrome and the Airship Mooring Mast; to the works of the Curtiss-Reid Company; the Canadian Vickers; and various aerodromes.

He met the following members of the Society: Mr. A. J. Berlyn, A.F.R.Ae.S.; Mr. J. A. McCullock, A.M.I.Ae.E.; Mr. R. P. Key, A.F.R.Ae.S.; Mr. R. J. Moffett, A.F.R.Ae.S., A.M.I.Ae.E.; Comdr. Pressy, in charge of the mast at St. Hubert; Mr. W. T. Reid, F.R.Ae.S. (who gave much valuable advice and help on the formation of branches); and Captain Sparks.

The Secretary also had the opportunity of talking with Captain Beck, of the Curtiss-Read Company, and Mr. J. A. McCurdy, of the same Company, and the pioneer pilot of Canada; Colonel Mulock of Canadian Airways, and many others.

In Toronto visits were made to a number of aerodromes and to Messrs. De Havilland's works under the guidance of Major R. Nicoll, of the Slotted Wing Corporation, and Mr. R. A. Loader, General Manager of De Havillands in Canada. In Toronto the Secretary had the opportunity of meeting Captain Roy Maxwell, and studying the excellent work being done by the Ontario Provincial Air Service and of meeting the late Capt. Charles Sutton, A.M.I.Ae.E., formerly of Dominion Explorers, Ltd., and of learning first-hand his experiences of flying in the northern area of Canada; Captain McNabb Hand, A.R.Ae.S.I., President of the Toronto Flying Club; Mr. F. Y. McEachren, President of the Slotted Wing Corporation; Major Geoffrey O'Brien, Test Pilot of De Havillands and others.

Major Lester Gardner, A.F.R.Ae.S., had charge of the Secretary's visit to New York and Washington. It was due to the great kindness of Major Gardner, who gave up practically all his time and used all his wide influence on behalf of the Secretary while the latter was in New York, that the Secretary had an opportunity of meeting many of the leaders in American aviation, and many members of the Society. Many of the latter were met at a special dinner arranged by Major Gardner to the Secretary. Among those met in New York or Washington were Major E. E. Aldrin, A.F.R.Ae.S., General Manager of the Standard Oil Company of New Jersey; Mr. Bernt Balchen, the pilot of the Byrd Antarctic Expedition; F. P. H. Beadle, A.F.R.Ae.S., of the Fairchild Manufacturing Corporation; Colonel Harry M. Blee, of the Department of Commerce, Washington; Rear-Admiral R. E. Byrd, U.S.N. (Ret.), Commander of the Byrd Antarctic Expedition; Mr. Glen H. Curtiss; the Hon. F. Trubee Davison, Assistant Secretary of War for Aeronautics, U.S.A.; Colonel E. A. Deeds; Dr. Techn. V. T. Friedrich, M.I.Ae.E.; the Hon. W. Irving Glover, Second Assistant Postmaster-General, U.S.A.; Captain R. J. Goodman Crouch, A.F.R.Ae.S.; Commander J. C. Hunsaker, Hon. Fellow; the Hon. David S. Ingalls, Assistant Secretary of the Navy for Aeronautics, U.S.A.; Professor A. Klemin, Professor of Aeronautical Engineering, Daniel Guggenheim School of Aeronautics, New York University; Mr. Charles L. Lawrence, F.R.Ae.S., of the Wright Aeronautical Corporation; Dr. Lewis, Secretary of the N.A.C.A., Washington; Mr. Jerome Lederer, A.M.I.Ae.E., Chief Engineer, Aero Engineering and Advisory Service, New York; Mr. W. B. Mayo, of the Ford Aeroplane Co.; Mr. Humphrey F. Parker, A.F.R.Ae.S.; Mr. F. B. Rentschler, President, Aeronautical Chamber of Commerce of America; Mr. Calvin Rice, Secretary of the American Society of Mechanical Engineers; Comdr. J. H. Towers, of the Navy Department, Washington; Mr. R. H. Upson, M.I.Ae.E., the designer of the well-known metal-clad airship; Professor E. P. Warner,

F.R.Ae.S., Editor of *Aviation*; Mr. T. P. Wright, A.F.R.Ae.S., Chief Engineer of the Curtiss Wright Corporation.

In New York and Washington the Secretary visited a number of aerodromes and works and had the opportunity of watching at the experimental field, near Washington, the methods of blind flying and radio beacon research which were being carried out. He also was able, thanks to the courtesy of Admiral Moffet, to visit Lakehurst and inspect the Los Angeles, the metal-clad airship and the movable mooring mast.

Everyone whom the Secretary met in Canada or the United States not only went out of their way to give him every possible facility to see and learn everything he wished, but they extended to him a hospitality which was at times embarrassing. The prestige of the Society is high in both countries, and it is to the Society that members and others are looking for providing the dissemination of reliable technical information. In the course of his visit the Secretary acquired much valuable information and was able to make arrangements for the regular supply of technical information from Canada and the United States to the Society which will be invaluable. The full and detailed report of the Secretary's visit is too long to be printed in the Journal, but a copy is on file in the offices of the Society.

The concrete results of the visit are reflected in the increased number of applications for membership from Canada and the United States; the formation of branches in Canada; and the establishing of personal relationship of a kind which will prove of inestimable value in the future.

Amended August 12th, 1930.

DRAFT of suggested Rules for Aeronautical Sections of Branches of the Engineering Institute of Canada, intended to constitute an Agreement between the Council of the Institute and the Council of the Royal Aeronautical Society.

1. Object of Aeronautical Sections.—Any Branch of The Engineering Institute of Canada may form an Aeronautical Section for the purpose of advancing aeronautical science and engineering, and more particularly for promoting that species of knowledge which distinguishes the profession of aeronautics.

2. Constitution.—Such an Aeronautical Section shall consist of—

(*a*) Corporate Members and Juniors of The Engineering Institute of Canada who apply for membership therein;

(*b*) Technical members of The Royal Aeronautical Society who apply for membership therein; and

(*c*) Such other persons as the Executive Committee of the Branch may see fit to elect.

3. Membership in an Aeronautical Section shall confer no technical or professional status whatever either in The Engineering Institute of Canada or in The Royal Aeronautical Society, nor shall such membership entitle the holder to represent himself as a member of The Engineering Institute of Canada or of The Royal Aeronautical Society.

4. The officers of an Aeronautical Section shall be appointed by the Executive Committee of the Branch, and shall be chosen so as to include as far as possible an equal number of corporate members of The Engineering Institute of Canada and technical members of The Royal Aeronautical Society.

5. The officers of an Aeronautical Section shall submit to the Executive Committee of the Branch an annual report of its activities, for inclusion in the annual branch report to the Council of The Engineering Institute of Canada, and a copy of the Section's annual report shall be forwarded by the General Secretary of The Engineering Institute of Canada to the Secretary of The Royal Aeronautical Society for the information of the Council of that body.

6. The annual subscription, which shall be payable by all members of an Aeronautical Section who are not corporate members or Juniors of the E.I.C., and/or technical members of the R.Ae.S., shall be fixed by the Executive Committee of the Branch, but shall in no case exceed five dollars.

7. The officers of an Aeronautical Section shall be responsible for the arrangements for the obtaining and preparation of papers, lectures, or communications on aeronautical subjects to be presented before the Section, and the Councils of The Engineering Institute of Canada and The Royal Aeronautical Society will render assistance in making these arrangements. Copies of all papers read before an Aeronautical Section shall be submitted to the Editor of *The Engineering Journal* for publication. Suitable papers presented before the Aeronautical Sections shall be published in the special section of *The Engineering Journal* as far as space permits.

8. A special section of *The Engineering Journal* will be devoted to aeronautical engineering, and this portion will be reprinted as a separate pamphlet monthly. The Secretary of The Engineering Institute of Canada will forward to the Secretary of The Royal Aeronautical Society each month a sufficient number of copies of this pamphlet for distribution to all members of that Society, and in return the Secretary of The Royal Aeronautical Society will forward a number of copies of The Royal Aeronautical Society *Journal* for all corporate members and Juniors of The Engineering Institute of Canada who have joined an Aeronautical Section.

9. The total cost of running off the extra copies of the pamphlet of The Engineering Institute of Canada and that of The Royal Aeronautical Society shall be shared equally by the two bodies.

10. The Council of The Engineering Institute of Canada and the Council of The Royal Aeronautical Society will respectively provide each Aeronautical Section (on request) free of charge with copies of *The Engineering Journal* and The Royal Aeronautical Society *Journal*, in the proportion of one copy of each for each twenty-five members of the Section, for circulation among the members.

11. The Council of The Engineering Institute of Canada will arrange for the examinations of The Royal Aeronautical Society in Canada (on request), to be held under the same conditions as the Institute's examinations. Invigilations will be in the hands of a Technical Member of The Royal Aeronautical Society; failing whom, a corporate member of The Engineering Institute of Canada may act. Papers will be sent out from England, and the candidates' answers returned there, under seal. The invigilator's fee of five dollars will be paid by The Royal Aeronautical Society.

12. An Aeronautical Section may frame and adopt such rules and by-laws as may be required for the regulation of its affairs, provided that these are not in conflict with those of The Engineering Institute of Canada, The Royal Aeronautical Society, the Branch of which the Section forms part, or with the present rules and agreement.

13. An Aeronautical Section of a Branch of The Engineering Institute of Canada, organised in accordance with the above requirements, will be recognised as a Canadian local Section of The Royal Aeronautical Society, and may be

referred to either as the Aeronautical Section of The Toronto (or Ottawa, etc., or as the case may be) Branch of The Engineering Institute of Canada, or as The Toronto (or Ottawa, etc.) Section of The Royal Aeronautical Society.

14. All the foregoing provisions are subject to annual reconsideration by the Councils of The Engineering Institute of Canada and The Royal Aeronautical Society, and this agreement shall be terminable by one year's notice on either side.

15. The above clauses, numbered one to fourteen, governing the formation and operation of Aeronautical Sections of Branches of The Engineering Institute of Canada and their recognition as Canadian Sections of The Royal Aeronautical Society, are hereby agreed upon and approved, as authorised by a resolution of the Council of The Engineering Institute of Canada dated.....
and a resolution of the Council of The Royal Aeronautical Society dated
.....

REVIEW

Marine Aircraft Elementary Naval Architecture

By Captain P. H. Sumner. (Crosby Lockwood and Son. 16/-.)

A book on flying boat design and construction is needed, and the time has come to write it. Captain Sumner realised this and has attempted to meet the demand. He has, however, made a very poor job of it. His book is a mass of vague and confused thinking. It is full of mistakes and shows every sign of having been written in a hurry.

In his introductory chapter, which takes up more than half the book, the author jumps from descriptions of the Saunders-Roe " Cutty Sark " and " Cloud " of 1930 to the Porte boats of 1915-16, from the Short " Cockle " to the Fairey " Atalanta " and back by way of the " Kingston " to the Short " Singapore."

In Chapter II we are told that " Lateral stability concerns rolling as a ship does in a beam sea." This wonderful sentence is typical of the author's definitions. In writing of certain continental designs, on page 111 he says: " The exceptional wide chord employed gives an aspect ratio of less than 8 compared with the more normal 10 or 12 and the load factor has been reduced to about 5 compared with a usual 6 or 7."

In Chapter III an explanation of Simpson's Rules is attempted. Whilst saying that, in the first rule, the odd ordinates must be multiplied by 2, Captain Sumner does not add that the even ordinates must be multiplied by 4.

In Chapter VI we are told that in addition to the frames, the boat-built hull is braced transversely by numerous timbers of small section spaced between the frames in accordance with best launch or yacht building practice—a statement which does not help those who come to this book to learn.

Apart from the initial letter, the index is not alphabetically arranged. This can be taken as an indication of the author's laziness.

Not even the type-setting, paper and binding justify the price of sixteen shillings.